Second round
Dutch Mathematical Olympiad

Friday 10 March 2023

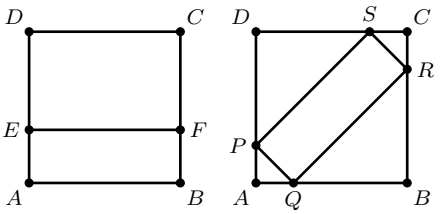
- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler, set square, and of course your mental skills.
- Good luck!

B-problems

For the B-problems you only have to give the answer (for example, a number). No explanation is required. A correct answer is awarded 4 points. For a wrong or incomplete answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact and reduced form, like $\frac{11}{81}$, or $2+\frac{1}{2}\sqrt{5}$, or $\frac{1}{4}\pi+1$, or 3^{100}.

B1. A teacher makes a big fraction with the numbers 1 up to and including 12. She writes some of the numbers in the numerator and puts $\times$-signs between them. She puts the remaining numbers in the denominator, again with $\times$-signs between the numbers. In both the numerator and denominator, there is at least one number. She makes sure that the fraction equals an integer that is as small as possible.
What is this integer?

B2. In the left square, rectangle $ABFE$ is drawn. In the right square, rectangle $PSRQ$ is drawn, where the sides QR and PS are parallel to the diagonal AC of the square. The sides of both squares have length 6. It turns out that the rectangles $ABFE$ and $PSRQ$ have exactly the same dimensions.
What is the length of AE?

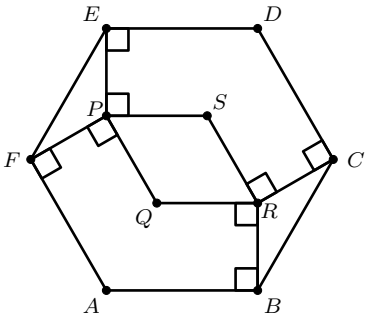


B3. The crab of a positive integer is the number you get when you write down its digits in reverse order. For example, the crab of 8267 equals 7628 and the crab of 15620 equals 2651 (because the leading zero is always dropped).
What is the smallest positive integer n such that n minus the crab of n equals 12345678?

B4. For the second round of the Mathematical Olympiad, 999 students are invited. Melanie creates invitation letters in order of participant number: $1, 2, 3, \ldots$. For some values of $n \geqslant 100$, she notices the following: the number of participant numbers from 1 up to and including n ending in a 5 is exactly equal to the integer formed by the last two digits of n.
For how many values of n (with $100 \leqslant n < 1000$) does this hold?

PLEASE CONTINUE ON THE OTHER SIDE

B5. The area of the regular hexagon $ABCDEF$ on the right is 6. Moreover, all the marked angles are right angles.
What is the area of the quadrilateral $PQRS$?



C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning that shows the correctness of your answer. A correct and well-explained answer is awarded 10 points. Partial solutions may also be worth some points. Therefore, write everything down clearly and hand in your drafts.
ATTENTION: Use separate sheets of paper for each C-problem and also hand in the drafts for each problem separately.

C1. Sara has 10 blocks numbered 1 to 10. She wants to stack all the blocks into a tower. A block can only be put on top of a block with a higher number, or on top of a block with a number that is exactly one lower. An example of such a tower is, from top to bottom: 2, 1, 5, 4, 3, 6, 7, 9, 8, 10. How many different towers are possible?

C2. Two positive integers having difference 20 are multiplied with each other; then 23 is added to the result.

(a) What is the smallest possible outcome that ends in 23? *Give this outcome (and the two corresponding integers with difference 20) and prove that no smaller outcome is possible.*

(b) Is it possible that the result is the square of an integer? *Give an example (and show that it is an example) or prove that this is impossible.*