Junior Wiskunde Olympiade
Problems part 2

Saturday 22 June 2024
Vrije Universiteit Amsterdam

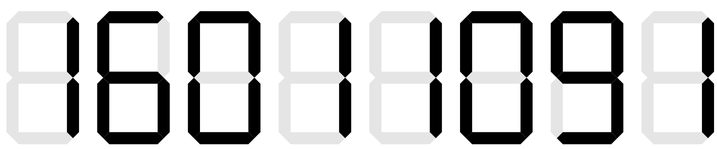
- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct and complete answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is also allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**

1. Lucas paints the entire outside of a cube blue. He then saws the cube into 27 equally sized cubes. He neatly stacks these 27 cubes so that he gets a tower of $27 \times 1 \times 1$ cubes.
At most how many of the 110 side faces of cubes on the outside of his tower are blue?

2. In a row there are 2024 people, numbered 1 to 2024, and each of them either always tells the truth or always lies. Moreover, all 2024 people know from each other whether they are always telling the truth or always lying. At some point, for each number n, the person numbered n makes the statement: "At least n of these people always lie."
How many people always tell the truth?

3. Joah has a number of large pots with marbles in them. At the beginning of the week, all pots contain a different positive number of marbles. On the first day of the week, he adds one marble to each pot. On the second day, he adds a marble to all pots whose number of marbles is divisible by 2. On the third day, he adds a marble to all pots whose number of marbles is divisible by 3. He continues like this until the seventh day. Then it turns out that he has several pots with exactly 50 marbles in them.
What is the maximum number of pots with exactly 50 marbles that Joah could have?

4. If you write the date 16 January 1091 with 8 digital digits in a row, it looks like this:

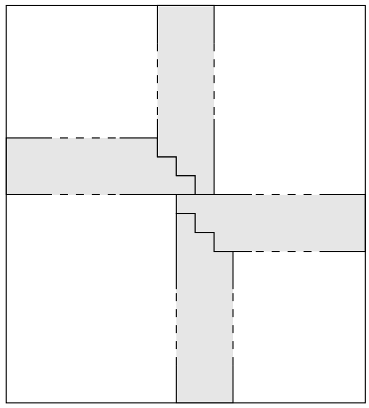


When you read this upside down, it reads as the exact same date.
What is the first date in the future (22 June 2024 or later) for which it is also true that, written in 8 digital digits consecutively, the date is exactly the same when read upside down?

PLEASE CONTINUE ON THE OTHER SIDE

5. The 'staircase figure' below has a height of 3 and steps that are each 1 wide and 1 high. We do not know how long the figure is: so the drawing is not to scale.

We put four such figures, all exactly the same size, in a rectangle as below. This drawing, too, is not to scale.



The rectangle has area 224 and the width of the top vertical staircase figure overlaps with $\frac{2}{3}$ of the width of the bottom staircase figure.
What is the area of one staircase figure?

6. What is the smallest integer $a > 10$ for which:

- a is divisible by 10 and
- $a + 1$ is divisible by 11 and
- $a + 2$ is divisible by 12?

7. The uppercase letters A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z can be written using only straight line segments. For the I, one line segment is enough; for the E, four line segments are needed. We call a sequence of at least two of these letters a word; so it does not have to be an existing word or even pronounceable. For example, FK is a word, and you write this word with six line segments.
How many words exist that can be written in uppercase letters with a total of exactly four line segments?

8. Roos has a chess board, a ruler, and a marker. She now chooses two vertices on the edge of the board so that when she draws the straight line between those two points with her marker, the board is divided into two parts. Here, for example, you can see how Roos divides the board into a part with area 49 and a part with area 15.
How many different values can the area of such a part have?