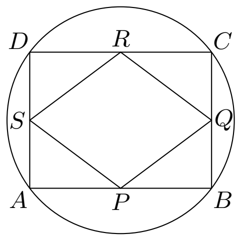
Junior Wiskunde Olympiade
Problems part 1

Saturday 22 June 2024
Vrije Universiteit Amsterdam

- The problems in part 1 are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form.
- A correct answer is awarded 2 points, for a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is also allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to finish these problems. **Good luck!**

1. A rectangle $ABCD$ has a circumscribed circle with radius 5. Points P, Q, R and S are the midpoints of the sides of rectangle $ABCD$.
What is the perimeter of quadrilateral $PQRS$?



A) 12 B) 14 C) $8\sqrt{5}$ D) 20 E) not uniquely determined

2. How many zeros does the number

$$2^{3^5} \times 3^{5^2} \times 5^{2^3}$$

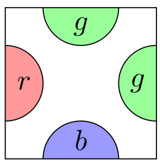
end with? In this problem, a^{b^c} is the number you get by first calculating the power

$$b^c = \underbrace{b \times b \times \ldots \times b}_{c \text{ times a } b}$$

and then raising a to this power.

A) 6 B) 8 C) 25 D) 243 E) 256

3. On a square puzzle piece, four semicircles are drawn, each coloured in red, green and blue as shown here (also indicated here with letters for clarity). The back of the puzzle piece is not coloured. You have four of these puzzle pieces (all coloured exactly as shown here) that you are allowed to shift and/or rotate. You place these in a 2×2-square against each other, such that semicircles that are adjacent have the same colour.



How many different colour patterns can you make in the 2×2-square? (For this problem, two such patterns that differ from each other by a rotation are considered to be different.)

A) 1 B) 2 C) 4 D) 5 E) 6

4. How many pairs of positive integers a and b are there, with $a > b$, both smaller than 100, and for which $a + b = (a - b)^3$?

A) 2 B) 3 C) 4 D) 5 E) 6

5. We are looking at numbers for which two adjacent digits always add up to the digit immediately to their right (if it exists). So two adjacent digits when added together are always smaller than 10, except possibly the last two digits. Furthermore, the first digit cannot be 0. An example of such a number is 1347, because $1 + 3 = 4$ and $3 + 4 = 7$.
How many such five-digit numbers do there exist?

A) 1 B) 2 C) 3 D) 5 E) 8

PLEASE CONTINUE ON THE OTHER SIDE

6. Inaya puts 88 fours in a row and gets a very big number:

$$\underbrace{4444\cdot\cdot\cdot\cdot4444}_{\text{88 fours}}$$

She writes the fours as a plus sign with a slash in the upper left corner. When she removes that slash, she gets a plus sign. So she can make additions like

$$4+44444+44+\cdot\cdot\cdot\cdot4+4+444$$

The addition may not begin or end with a plus sign. By cleverly choosing at which of the 88 fours she removes the slash, Inaya makes an addition with the result being 4444.
In how many fours did she remove the slash?

A) 4 B) 11 C) 22 D) 29 E) 30

7. We call a positive integer a *reverse difference* if it can be written as a positive integer whose last digit is not a 0, minus the number consisting of the same digits in reverse order. For example, 2178 is a reverse difference because $4202 - 2024 = 2178$.
Which of the following numbers is not a reverse difference?

A) 1359 B) 2538 C) 3906 D) 4447 E) 5355

8. On a one-way road with two lanes, there are blue and red cars driving. The left lane contains only blue cars, the right lane only red. At a certain point, the cars have to merge to one lane. In doing so, the blue cars are a bit bolder than the red ones, sometimes causing several blue cars to end up behind each other, but never causing two red cars to end up behind each other. A little further on, the cars have to wait at a traffic light.
If you look at the first ten cars at the traffic light, how many possible colour combinations can occur? (So for example bbbbbbbbbb or rbrbrbrbrb.)

A) 89 B) 100 C) 144 D) 233 E) 1024