

Junior Wiskunde Olympiade

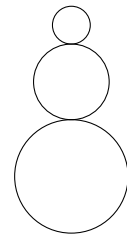
Problems part 2



Saturday 3 June 2023
Vrije Universiteit Amsterdam

- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct and complete answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**

1. Tim has exactly six meters of iron wire. He cuts the iron wire into a number of pieces, in such a way that each piece is an integer number of meters long. From each of those pieces, he makes a circle. Then he stacks those circles perfectly vertically balancing on top of each other. A possible front view is shown in the figure to the right.

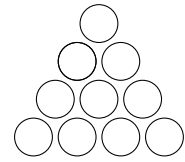


How many possible heights can his structure have?

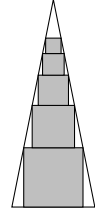
2. In how many ways can you divide the numbers 1 up to and including 10 into pairs such that, for each pair, the largest number is at least twice the smallest number?
3. A city wants to conduct a trial with alternative time measurements: a 24-hour day will be divided into 15 howers, an hower into 72 minotes, and a minote again into 80 seconds, which are as long as seconds in ordinary timekeeping. In other words, 15 howers of 72 minotes with 80 seconds last as long as 24 hours of 60 minutes with 60 seconds. In the city, digital clocks are used on which the first two digits indicate howers, the next two indicate minotes and the last two indicate seconds. The time 00 : 00 : 00 is at midnight, just like an ordinary clock. At a given moment, such an alternative clock reads 10 : 10 : 10. What time (hh : mm : ss) is displayed on an ordinary digital clock at that moment?
4. Dotty has drawn a paper with six dots on it, no three of which lie on a line. With a pen, she draws a straight line from a certain dot to another dot. Then, without lifting her pen, she draws a straight line to a dot she has not visited before. She continues like this until she has visited all the dots. Finally, she draws a straight line from the last dot to the first dot. How many figures can Dotty make this way?
5. We consider the number $666 \dots 666$ which consists of 2023 sixes. The square of this number has 4046 digits. How many of those digits are a 5?

PLEASE CONTINUE ON THE OTHER SIDE

6. A triangular number is the number of circles you can place in an equilateral triangle of a certain height. For example, in the figure on the right, you can see that the fourth triangular number is 10. Jasmine writes down the first 1000 triangular numbers: 1, 3, 6, 10, 15, ...
How many of these numbers end in a 0?



7. Five squares are stacked on top of each other to fit exactly into an isosceles triangle, see the figure on the right (not drawn to scale). The largest square has sides of length 3, the smallest square has sides of length $\frac{4}{3}$.
What is the length of the side of the middle square?



8. Ikram has a large bowl with little balls in it. On each ball a positive integer is written. If he randomly picks three balls from the bowl and takes the difference between the largest and smallest number on these three balls, it turns out that the outcome is always also on one of the balls in the bowl (or on one of the three balls he just picked up). He always puts the balls back in the bowl. Irkam knows for sure that there are balls with 3, 6 and 2023 in the bowl.
At least how many balls are in the bowl?