Junior Wiskunde Olympiade
Problems part 1

Saturday 30 September 2017
Vrije Universiteit Amsterdam

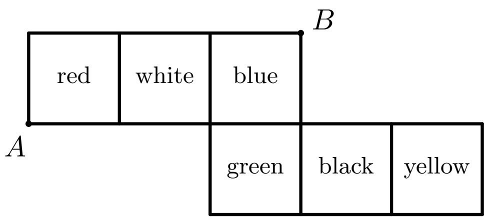
- The problems in part 1 are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form.
- A correct answer is awarded 2 points, for a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to finish these problems. **Good luck!**

1. A positive three-digit number is called *nice* if the sum of the last two digits equals the first digit. For example, 123 is *not* nice, because 1 is not equal to $2+3$.
How many three-digit numbers are nice?
Note that a three-digit number cannot start with digit 0.

A) 45 B) 48 C) 50 D) 54 E) 55

2. The faces of a cube have different colours. In the figure you can see a net for the cube. The points A and B in the net correspond to two vertices of the same face of the cube.
What colour does that face have?

A) red B) blue C) green D) black E) yellow



3. We consider sequences of 20 integers. The integers can be positive or negative, but cannot be zero. Also, we impose the following conditions on the sequences: of any two adjacent numbers at least one is positive; the sum of any three adjacent numbers is negative; the product of any four adjacent numbers is positive.
Consider the following four statements about such sequences:

- There can never be two adjacent positive numbers.
- There may be more positive than negative numbers.
- The sum of all 20 numbers is always negative.
- The number -1 can never occur.

How many of these statements are true?

A) 0 B) 1 C) 2 D) 3 E) 4

PLEASE CONTINUE ON THE OTHER SIDE

4. The number $n^2 + 21$ is the square of an integer.
For how many positive integers n does this hold?

A) 0 B) 1 C) 2 D) 3 E) 4

5. Sanne is building a $9\times9\times9$ cube by gluing $1\times1\times1$ blocks together. She doesn't have quite enough blocks to complete the task. Therefore, she decides to leave out some of the blocks from the large cube. In order to still get a nice rigid cube, she makes sure that no two holes (left out blocks) are adjacent. In fact, two holes should not even touch in an edge or a single vertex. Also, she does not leave out any of the blocks on the outside of the cube.
What is the minimum number of blocks that Sanne needs to build the cube?

A) 365 B) 604 C) 665 D) 673 E) 702

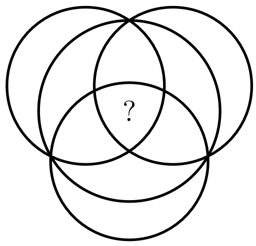
6. Peter starts out with the numbers 1, 2, 3, and 4. He may take two of his numbers and replace them by their sum, their product, or their difference. He performs this replacement step three times, after which a single number remains.

Example. He could replace the 2 and the 4 by $2 + 4 = 6$, then replace the 1 and the 3 by $3 - 1 = 2$, and finally replace the 6 and the 2 by $8 = 2 + 6$. Then, the remaining number would be 8.

Which of the following five numbers *cannot* be the number that remains?

A) 28 B) 30 C) 32 D) 34 E) 36

7. Four circles together enclose ten regions in the plane, as in the figure. We want to place the numbers 1 to 10 inside the regions (one number per region). This must be done in such a way that adding the numbers inside a circle gives the same answer for all four circles.
Which number should be placed in the region with the question mark?



A) 1 B) 2 C) 4 D) 6 E) 7

8. You have a collection of hats. Each hat has three attributes: the colour (red or blue), the shape (top hat or pointed hat), and the pattern (spots or stripes). You put a number of gnomes in a room and put a hat on each of them. For any two gnomes their hats must be different, yet share at least one attribute (for example: both hats are blue). The gnomes can see everyone's hat, except their own. The gnomes are not allowed to communicate with one another.
What is the minimum number of gnomes you have to put in the room in order to be be sure that one of them can determine one of the attributes of his own hat?

A) 3 B) 4 C) 5 D) 8 E) That is impossible for any number of gnomes.