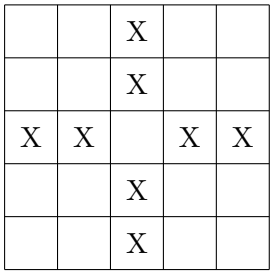
Final round
Dutch Mathematical Olympiad

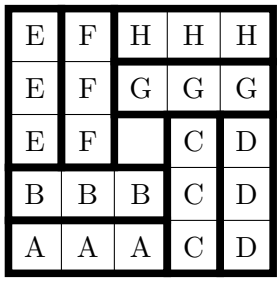
Friday 12 September 2025

Solutions

1. You can always find a prize by scratching open the eight boxes where there is a cross in the figure at the bottom left. Indeed, it it is easy to verify that each of the 30 possible 3×1-rectangles overlaps with at least one of the boxes marked with a cross.

On the other hand, it is really necessary to scratch open eight boxes. In the figure below right, you can see eight possibilities for the three prize boxes: A to H. If seven or fewer boxes were scratched open, then in at least one of these eight possible situations you would not have found a prize.





2. Version for klas 4 and below

(a) If we substitute $n = 1, \ldots, 5$, we see that on the left side we get the numbers $1, 0, 0, 12$ and 100 respectively. The number on the right-hand side is a positive multiple of 4, so 1 and 0 are not possible. Furthermore, we see that $12 = 4 \cdot 3$ and 3 is not a square. Finally, we see that $100 = 4 \cdot 25$ and 25 is the square of the prime number $p = 5$. So the only solution with $1 \leqslant n \leqslant 5$ is $(n, p) = (5, 5)$.

(b) Suppose (n, p) is a solution with $n \geqslant 6$. First, we note that

$$n! - (n-1)n = ((n-2)! - 1)(n-1)n$$

is divisible by $n-1$ and n. Only one of those two numbers can be divisible by p, because their difference, which is 1, is not divisible by p. The divisors of $4p^2$ are $1, 2, 4, p, 2p, 4p, p^2, 2p^2$ and $4p^2$ and only the divisors 1, 2 and 4 are not divisible by p (except if $p = 2$, then only the divisor 1 remains). So that means $n - 1$ or n must equal 1, 2 or 4. In particular, it follows that n is at most 5, which is a contradiction.

2. Version for klas 5 & klas 6

In the solution for klas 4 and below, part (a) investigates all solutions with $n \leqslant 5$. In part (b), it is proven that there are no solutions with $n \geqslant 6$.

3. We will prove that the result of this addition is always 900.

In each square, write two more numbers: in yellow the total number of squares already filled in the row of that square, and in red the total number of squares already filled in the column of that square. The number Amber writes down is then the yellow number plus the red number. If you look at the yellow numbers per row, you will see that you get the numbers 0 up to 9: the first square chosen gets a 0, the next one a 1, and so on. Similarly, you get the red numbers 0 to 9 per column. So the sum of the yellow numbers in a row (and the red numbers in a column) is

$$0 + 1 + 2 + \ldots + 9 = 45.$$

In total, there are 10 rows and 10 columns. So the total sum is always $20 \cdot 45 = 900$.

4. Version for klas 4 and below

In triangles ADP and CDA, two angles are equal and so the triangles are similar. It follows that $\frac{|AD|}{|DP|} = \frac{|CD|}{|AD|}$. Similarly, triangles BDP and CDB are similar and we find that $\frac{|BD|}{|DP|} = \frac{|CD|}{|BD|}$. Combining all this, we find that

$$|AD|^2 = |CD| \cdot |DP| = |BD|^2.$$

So we find that $|AD| = |BD|$ and thus D is the midpoint of the line segment AB.

4. Version for klas 5 & klas 6

Suppose D is the intersection of CP and AB. In the solution for klas 4 and below it is proven that D is the midpoint of the line segment AB. This implies that CP is the median of triangle ABC through C.

5. Version for klas 5 & klas 4 and below

(a) A possible solution for odd n is as follows. Put pawn 0 at vertex 0 and for every $i = 1, \ldots, n-1$, put pawn i at vertex $n - i$.
If $0 < i \leqslant \frac{n-1}{2}$, then i is smaller than $n - i$ and pawn i, which starts at vertex $n - i$, must therefore first move i places to get to vertex 0 and then another i to end at the correct vertex. So the number of spots these pawns must move up is $2, 4$, and so on, up to $2 \cdot \frac{n-1}{2} = n - 1$. Pawn 0 must move up 0 spots. Altogether, those are exactly all possible even numbers.
If $\frac{n+1}{2} \leqslant i < n$, then $n - i$ is smaller than i and pawn i, starting at vertex $n - i$, must therefore move up $i - (n - i) = 2i - n$ steps clockwise to arrive at the correct vertex. So these pawns must move 1, 3, 5, and so on, up to $n - 2$, spots to arrive at the correct vertex. Those are exactly the odd numbers.
Other solutions are also possible: for example, you can also put pawn i at vertex $2i$ for $0 \leqslant i \leqslant \frac{n-1}{2}$, and at vertex $2i - n$ for $\frac{n+1}{2} \leqslant i < n$.

(b) Assume that we have a solution. We are going to show that n must be odd and therefore no solutions are possible if n is even. Suppose pawn i starts at vertex a_i. The distance to the correct vertex, vertex i, is then $\Delta_i = i - a_i$ if $i \geqslant a_i$, and $\Delta_i = (n + i) - a_i$ if $i < a_i$. On the one hand, the pawns all start at different vertices and so we have that

$$\sum_{i=0}^{n-1} a_i = \sum_{j=0}^{n-1} j.$$

On the other hand, the pawns also all end at different vertices:

$$\sum_{i=0}^{n-1} i = \left(\sum_{i=0}^{n-1} (a_i + \Delta_i) \right) - cn,$$

where c is the number of i for which $i < a_i$. In particular, it therefore follows that

$$\sum_{i=0}^{n-1} \Delta_i = cn.$$

If the Δ_i are all distinct, then they are exactly the numbers 0 up to $n - 1$. So we then find that $0 + 1 + \ldots + (n - 1) = cn$, or equivalently $\frac{1}{2}n(n - 1) = cn$. This implies that $\frac{1}{2}(n - 1) = c$, so $n = 2c + 1$. Hence, n is odd, as desired.

5. Version for klas 6

In the version for klas 5 & klas 4 and below, it is proved in part (a) that this is possible if n is odd, and in part (b) it is proved that it is impossible if n is even.