Finale
Dutch Mathematical Olympiad

Version klas 6

Friday 14 September 2018
Technische Universiteit Eindhoven

- Available time: 3 hours.
- Each problem is worth 10 points. Points can also be awarded to partial solutions.
- Not only the answer is important; you also have to write down a clear reasoning that shows correctness of your answer.
- Calculators and formula sheets are not allowed. You can only bring a pen, ruler (set square), compass and your math skills.
- Use a separate sheet for each problem and also hand in your draft sheets (for each problem separately!). Good luck!

1. We call a positive integer a *shuffle number* if the following hold:

(1) All digits are nonzero.

(2) The number is divisible by 11.

(3) The number is divisible by 12. If you put the digits in any other order, you again have a number that is divisible by 12.

How many 10-digit shuffle numbers are there?

2. The numbers 1 to 15 are each coloured blue or red. Determine all possible colourings that satisfy the following rules:

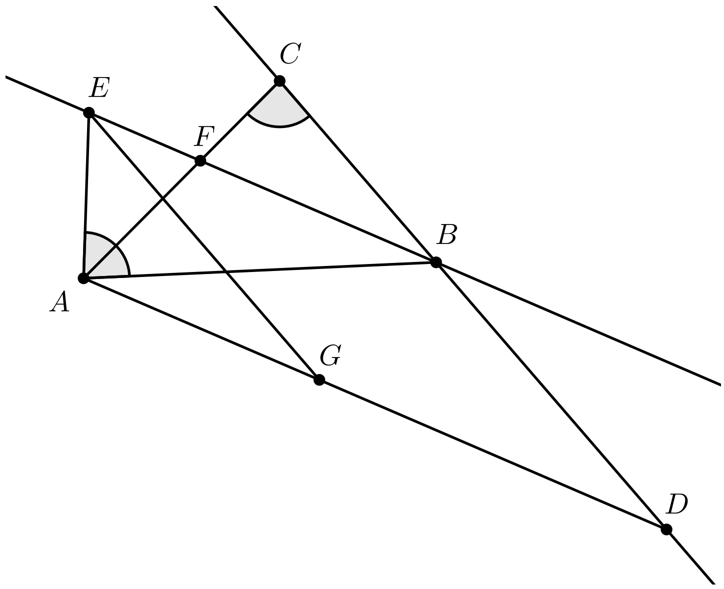
- The number 15 is red.
- If numbers x and y have different colours and $x + y \leqslant 15$, then $x + y$ is blue.
- If numbers x and y have different colours and $x \cdot y \leqslant 15$, then $x \cdot y$ is red.

3. Determine all triples (x, y, z) consisting of three *distinct* real numbers, that satisfy the following system of equations:

$$\begin{aligned} x^2 + y^2 &= -x + 3y + z, \\ y^2 + z^2 &= x + 3y - z, \\ x^2 + z^2 &= 2x + 2y - z. \end{aligned}$$

PLEASE CONTINUE ON THE OTHER SIDE

4. In triangle ABC, $\angle A$ is smaller than $\angle C$. Point D lies on the (extended) line BC (with B between C and D) such that $|BD| = |AB|$. Point E lies on the bisector of $\angle ABC$ such that $\angle BAE = \angle ACB$. Line segment BE intersects line segment AC in point F. Point G lies on line segment AD such that EG and BC are parallel.
Prove that $|AG| = |BF|$.



5. At a quiz show there are three doors. Behind exactly one of the doors, a prize is hidden. You may ask the quizmaster whether the prize is behind the left-hand door. You may also ask whether the prize is behind the right-hand door. You may ask each of these two questions multiple times, in any order that you like. Each time, the quizmaster will answer 'yes' or 'no'. The quizmaster is allowed to lie at most 10 times. You have to announce in advance how many questions you will be asking (but which questions you will ask may depend on the answers of the quizmaster). What is the smallest number you can announce, such that you can still determine with absolute certainty the door behind which the prize is hidden?