1. We consider positive integers written down in the (usual) decimal system. Within such an integer, we number the positions of the digits from left to right, so the leftmost digit (which is never a 0) is at position 1.

An integer is called even-steven if each digit at an even position (if there is one) is greater than or equal to its neighbouring digits (if these exist).

An integer is called oddball if each digit at an odd position is greater than or equal to its neighbouring digits (if these exist).

For example, 3122 is oddball but not even-steven, 7 is both even-steven and oddball, and 123 is neither even-steven nor oddball.

(a) Prove: every even-steven integer greater than 9 can be obtained by adding two oddball integers.

(b) Prove: there exists an oddball integer greater than 9 that cannot be obtained by adding two even-steven integers.

2. A parallelogram $ABCD$ with $|AD| = |BD|$ has been given. A point $E$ lies on line segment $BD$ in such a way that $|AE| = |DE|$. The (extended) line $AE$ intersects line segment $BC$ in $F$. Line $DF$ is the angle bisector of angle $CDE$.

Determine the size of angle $ABD$.

3. Six teams participate in a hockey tournament. Each team plays exactly once against each other team. A team is awarded 3 points for each game they win, 1 point for each draw, and 0 points for each game they lose. After the tournament, a ranking is made. There are no ties in the list. Moreover, it turns out that each team (except the very last team) has exactly 2 points more than the team ranking one place lower.

Prove that the team that finished fourth won exactly two games.
4. If we divide the number 13 by the three numbers 5, 7, and 9, then these divisions leave remainders: when dividing by 5 the remainder is 3, when dividing by 7 the remainder is 6, and when dividing by 9 the remainder is 4. If we add these remainders, we obtain $3 + 6 + 4 = 13$, the original number.

(a) Let $n$ be a positive integer and let $a$ and $b$ be two positive integers smaller than $n$. Prove: if you divide $n$ by $a$ and $b$, then the sum of the two remainders never equals $n$.

(b) We consider integers $n > 229$ having the following property: if you divide $n$ by 99, 132, and 229, then the sum of the three remainders is $n$. Prove that for such an integer $n$ the two remainders obtained when dividing $n$ by 99 and 132 add up to 229.

(c) Determine all integers $n > 229$ having the property that if you divide $n$ by 99, 132, and 229, the sum of the three remainders is $n$.

5. The eight points below are the vertices and the midpoints of the sides of a square. We would like to draw a number of circles through the points, in such a way that each pair of points lie on (at least) one of the circles. Determine the smallest number of circles needed to do this.

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