Second round
Dutch Mathematical Olympiad

Friday, March 13, 2026

Solutions

B-problems

B1. 16 Consider triangles ABP, ADP, BCP and CDP. If we take AP and CP as bases for those four triangles, then the triangles have the same height. We see that the areas of ABP (or ADP) and BCP (or CDP) thus relate as $|AP| : |CP|$. It follows from the data in the statement that this ratio is $2 : 1$. So that means that BCP and CDP together have exactly $\frac{1}{3}$ of the area of the rectangle. That area is $\frac{1}{3} \cdot 12 \cdot 8 = 32$. Triangles BCP and CDP have the same area. Triangle BCP thus has area 16.

B2. 35 In the table on the right you can see an example where the value 35 is reached. We are now going to show that this is the largest possible value. We do this by systematically checking what the maximum value is. First, we note that you always get the largest possible outcome by multiplying large numbers by large numbers and small numbers by small numbers. For example, if $a < b$ and $c < d$, then $ac + bd$ is greater than $ad + bc$; this is because their difference is $(ac + bd) - (ad + bc) = b(d - c) - a(d - c) = (b - a)(d - c)$ and this is positive. We are going to use this all the time. We now systematically run through the cases according to what the largest number in the bottom row is.

1	2	3	4	5
1	2	5	4	3
1	4	15	16	15

- The largest number is $5 \cdot 5$. We have two times 1, 2, 3, 4 left. By multiplying small numbers by small numbers and large numbers by large numbers, we get $1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4 = 30$ as the largest result.
- The largest number is $5 \cdot 4$ (or $4 \cdot 5$). We are then left with the numbers 1, 2, 3, 4 and 1, 2, 3, 5. The largest outcome is $1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 5 = 34$.
- The largest number is $4 \cdot 4$. We then have two times 1, 2, 3, 5 left, but $5 \cdot 5$ may not occur, because that is greater than $4 \cdot 4$. The largest result is now $1 \cdot 1 + 2 \cdot 2 + 3 \cdot 5 + 5 \cdot 3 = 35$.
- The largest number is $5 \cdot 3$ (or $3 \cdot 5$). We then have the numbers 1, 2, 3, 4 and 1, 2, 4, 5, but $4 \cdot 5$ and $4 \cdot 4$ may not occur. The largest result is now $1 \cdot 1 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 2 = 32$.
- The largest number is $4 \cdot 3$ (or $3 \cdot 4$). We are then left with the numbers 1, 2, 3, 5 and 1, 2, 4, 5, but $5 \cdot 5$, $4 \cdot 5$ and $3 \cdot 5$ may not occur. The largest result is now $1 \cdot 1 + 2 \cdot 5 + 3 \cdot 4 + 5 \cdot 2 = 33$.
- The largest number is $5 \cdot 2$ (or $2 \cdot 5$). We are then left with the numbers 1, 2, 3, 4 and 1, 3, 4, 5, but $5 \cdot 5$, $5 \cdot 4$, $5 \cdot 3$, $4 \cdot 4$ and $4 \cdot 3$ may not occur. So the 5 must be combined with 1 or 2 and both the fours as well. The largest result is then $1 \cdot 4 + 2 \cdot 5 + 3 \cdot 3 + 4 \cdot 1 = 27$.
- The largest number is $3 \cdot 3$. This leaves us with two times 1, 2, 4, 5. Now the 5 may only be combined with the 1. The only possible outcome is then $1 \cdot 5 + 2 \cdot 4 + 4 \cdot 2 + 5 \cdot 1 = 26$.
- The largest number is 8 or lower. Then the largest outcome is at most $8 + 8 + 8 + 8 = 32$ (and in practice much lower).

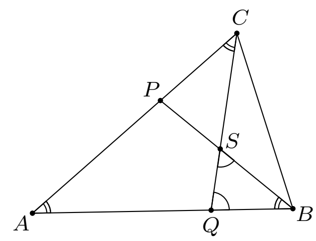
We see that 35 is indeed the maximum value.

B3. 156 If we start with 1 or 2, we are immediately done. If we start with a number between 3 and 12, we are done in two steps at most. The numbers 3 to 7 (except 5) go first to 5 and then to 1. The numbers 8 to 12 (except 10) go to 10 first and then to 2. From the numbers between 13 and 62 (except the multiples of five themselves), we first go to $15, 20, 25, \ldots,$ or 60 and after division by 5, we have a number between 3 and 12 as above. In this way, we can keep track of the last number in the sequence, as in the table below.

first number	last number	number of rows
1	1	1
2	2	1
3–7	1	5
8–12	2	5
13–37	1	25
38–62	2	25
63–187	1	125
188–200	2	13

We see that there are a total of $1 + 5 + 25 + 125 = 156$ sequences whose last number is a 1.

B4. $36°$ Triangles ABP and ACQ are isosceles, so $\angle ABP = \angle BAC = \angle ACQ$. We call that angle α. Triangle BSQ is also isosceles with top B, so $\angle BSQ = \angle SQB$. We call that angle β. These angles are marked in the picture on the right. Since the angles in triangle BSQ add up to $180°$, we have $\alpha + 2\beta = 180°$. At the same time, we see that $\angle AQC = 180° - 2\alpha$ because of the sum of the angles in triangle ACQ, but also that $\angle AQC = 180° - \beta$, because AQB is a straight angle. It follows that $\beta = 2\alpha$. Substituting that into the first equation, we get that $5\alpha = 180°$, or $\alpha = 36°$.



B5. 210 Every time Tim has two numbers a and b and replaces one of the numbers by $a + b$, nothing changes about the common divisors of the two numbers. If a and b were both divisible by a number d, then $a+b$ is also divisible by that. Conversely, if a and $a+b$ are divisible by d, then so is b (and the same with a and b vice versa).

At the beginning, the two numbers 1 and 1 have no common divisor except 1. Therefore, if N is a number that has a divisor greater than 1 in common with i for all $i = 2, 3, 4, \ldots, 10$, then it is not possible for Tim to get the number N on the board along with each of the numbers 2 to 10. The smallest number that satisfies this is $2 \cdot 3 \cdot 5 \cdot 7 = 210$; the number must have a divisor greater than 1 in common with the prime numbers 2, 3, 5 and 7 and must therefore be a multiple of 210, and the number 210 also has a common divisor greater than 1 with the remaining numbers 4, 6, 8, 9 and 10.

Thus, if we choose a smaller N, then there is one of the numbers 2, 3, 5 and 7, say i, which has no divisor greater than 1 in common with N. We can then work back exactly what must have been on the board. We start with N and i and each time replace the greater of the two numbers with the difference between them. In the end, we then get exactly the greatest common divisor of N and i, which is 1. For example, if you start with 30 and 7, you get 23 and 7, next 16 and 7, then 9 and 7, then 7 and 2, then 5 and 2, then 3 and 2, then 2 and 1, and finally 1 and 1. By reversing this row, you can see how Tim could get the numbers 30 and 7 on the board.

The reverse procedure described in the last paragraph is also called the Euclidean algorithm.

C-problems

C1. (a) There are lots of balanced squares for $n = 6$. An example is below.

1	2	3	4	5	6
2	3	4	5	6	1
3	4	5	6	1	2
4	5	6	1	2	3
5	6	1	2	3	4
6	1	2	3	4	5

We put the numbers 1 to 6 in a sequence and rotated this sequence by one position for each new row. Thus, the odd and even numbers alternate each time and in each 2×2-square there are exactly two even and two odd numbers and the sum is even.

(b) We are going to show that there is no balanced square for $n = 7$. Suppose such a square does exist. Look at the first column. Since the numbers 1 to 7 must all occur in that column, somewhere there is an even number below an odd number or vice versa. Suppose row i contains an even number and row $i + 1$ contains an odd number (see the letters 'e' and 'o' in the example below).

e	a_1	a_2	b_1	b_2	c_1	c_2
o	a_3	a_4	b_3	b_4	c_3	c_4

Next, look at the three 2×2-squares to the right of these two numbers. In the example, these are marked with the letters 'a', 'b' and 'c'. The sum of the numbers in each 2×2-square must be even. The sum of the two numbers in the first column, on the other hand, is odd. This means that the sum of all the numbers in the i-th and $(i + 1)$-th row together is odd. However, that sum is $2 \cdot (1 + 2 + \ldots + 7)$, which is even. This is a contradiction. Therefore, there is no balanced square for $n = 7$.

C2. (a) We consider where the line of sight from Esther's table to the top left table intersects the five rows in front of her (and her own row). If this is at two tables on consecutive rows, then she sees 5 tables. In particular, this is true if she sees a table in the row in front of her on the line of sight (because her own table and that table are in two consecutive rows). This is also the case if she sees a table 4 rows in front of her on the line of sight, because then she sees a table on the 4th and 5th row in front of her. If she sees a table 2 rows in front of her on the line of sight, then she also sees one 4 rows in front of her, so then we are also done. If she sees a table 3 rows in front of her on the line of sight, then also 6 rows in front of her (even if that row doesn't exist at all), so then she sees a table on the 5th and 6th rows in front of her and we are also done. The only case left is that she only sees a table in the 5th row in front of her, namely that corner table. So in that case, she sees only 1 table. Both options occur, for example if she is in the 6th and 2nd column respectively.

(b) We consider the line of sight from Esther's table to the top left table. Suppose Esther first sees a table on this line of sight k rows in front of her. We prove that she then also sees a table exactly on all multiples of k rows in front of her. Clearly, at least on all these multiples of k rows in front of her there is a table to be seen. And now suppose that an additional table could be seen on $ak + \ell$ rows in front of her by $0 < \ell < k$, then she would also see a table on this line of sight on ℓ rows in front of her as well, whereas she only saw the first table on row k (counting from her); contradiction.

Since she sees the corner table 10 rows in front of her, 10 must therefore be a multiple of k. It follows that $k = 1, 2, 5$ or 10 are the only options. So then she sees respectively 10, 5, 2 or 1 tables on that line. All four options occur, for example, if she is in the 11th, 6th, 3rd and 2nd column, respectively.

(c) Suppose Esther is sitting in a place with a rows in front of her and b rows behind her, and c columns to her left and d columns to her right. Assume, without loss of generality, that the line of sight to the upper left contains only 1 table (namely, only the corner table). Now consider the sight lines to the upper right and lower left. On them are 6 and 10, or 6 and 15, or 10 and 15 tables. First consider the case with 6 tables on the line of sight to the upper right and 10 tables on the line of sight to the lower left. Then we find that a (and d) are multiples of 6, and furthermore that c (and b) are multiples of 10. From this it follows that both a and c are also multiples of 2. But in that case, there are at least 2 tables on the line of sight to the upper left; there is also a table $\frac{a}{2}$ rows in front of Esther and $\frac{c}{2}$ columns to her left. This contradicts the assumption that there was only one table on this line of sight.

We find the same contradiction with 6 and 10 reversed. Similarly, for 6 and 15, it holds that there are at least 3 tables on the line of sight to the top left, since 6 and 15 are both multiples of 3. And for 10 and 15 the same is true; then there are at least 5 tables on the line of sight to the top left.

In all cases, we have a contradiction, so there is no such exam hall.