

First round

Dutch Mathematical Olympiad



22 January – 1 February 2018

- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$.
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 2 February on the website: www.wiskundeolympiade.nl.
- Good luck!

A-problems

1. In a classroom there are chairs and stools. On each chair and on each stool one child is seated. Each chair has 4 legs, each stool has 3 legs and each child has 2 legs. Together, we have a total of 39 legs.

How many chairs are there in the classroom?

- A) 3 B) 4 C) 5 D) 6 E) 9

2. On an island, there are knights and knaves. Knights always speak the truth and knaves always lie. On the island you meet five people. You know that four of them are knights and one of them is a knave, but you do not know who is the knave. They make the following statements about the island inhabitants:

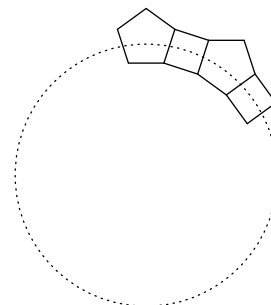
- A: "All knaves have shoe size 40."
- B: "All people with shoe size 40 have a goldfish."
- C: "All people with a goldfish are knaves."
- D: "I have shoe size 40."
- E: "I have a goldfish."

Which of them is the knave?

- A) A B) B C) C D) D E) E

3. If you continue the chain of squares and regular pentagons in the same way, does it connect to itself after going around? If so, how many pentagons do you need?

- A) 9 B) 10 C) 11 D) 12 E) It does not connect.



PLEASE CONTINUE ON THE OTHER SIDE

4. Julian wants to compose a list of integers. He wants the list to be as long as possible. Each integer on the list must consist of one or more of the digits 1 to 9. Moreover,
- each of the digits 1 to 9 is used exactly once;
 - no integer in the list is divisible by another integer in the list.

What is the maximum number of integers in Julian's list?

- A) 4 B) 5 C) 6 D) 7 E) 8

5. Nine people are at a party. While entering, some of them shook hands. Quintijn is at the party and asks each of the others how many hands they shook. He gets eight different answers. How many hands did Quintijn shake?

- A) 0 B) 1 C) 2 D) 3 E) 4

6. Birgit is studying positive integers n for which n is divisible by 4, $n + 1$ is divisible by 5, and $n + 2$ is divisible by 6.

How many of such integers n are smaller than 2018?

- A) 16 B) 17 C) 18 D) 33 E) 34

7. A frog starts in the point at coordinates $(0, 0)$ in the plane. He can make three kinds of jumps:

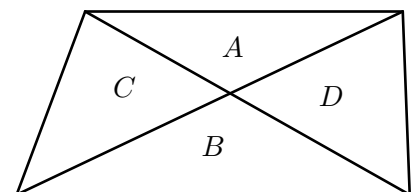
- from (x, y) to $(x, y - 5)$;
- from (x, y) to $(x - 2, y + 3)$;
- from (x, y) to $(x + 4, y + 9)$.

Ahead, there are three juicy snacks that the frog would like to eat: a worm at $(2013, 2018)$, a beetle at $(2018, 2019)$, and a snail at $(2018, 2023)$.

Which of these snacks can the frog reach?

- A) the worm and the snail B) the beetle and the snail C) the worm and the beetle
D) only the beetle E) only the snail

8. Harold draws a trapezium with parallel top and bottom sides. The length of the top side is smaller than the length of the bottom side. The two diagonals divide the trapezium into four triangles. The area of the upper triangle is called A , of the lower B , of the left C , and of the right D . An example of such a trapezium is depicted on the right.

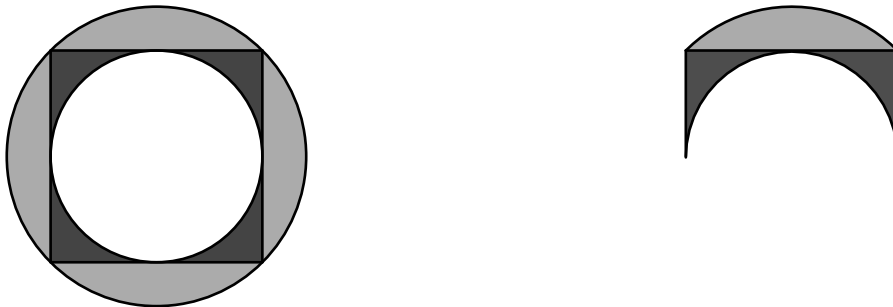


Which of the following equalities holds for any such trapezium?

- A) $A + C = B + D$ B) $A + D = B + C$ C) $A + B = C + D$
D) $A : B = D : C$ E) $A : C = D : B$

B-problems

1. Three years ago, Rosa's mother was exactly five times as old as Rosa was at that time. At that moment, Rosa's mother was just as old as Rosa's grandmother was when Rosa's mother was born. Now, Rosa's grandmother is exactly seven times as old as Rosa is.
How old is Rosa's mother now?
2. Nanda and Mike both have a note containing the same five-digit number. Nanda puts the digit 4 in front and the digit 8 at the end of her number to obtain a seven-digit number. Mike puts one digit in front of his number. Comparing their new numbers, Nanda's number turns out to be exactly 6 times as large as Mike's.
What was their starting number?
3. We consider a square, the circle through the vertices of the square and the circle touching the four sides of the square (see the left-hand figure). The ring-shaped area between the two circles is divided into four dark pieces (inside the square) and four light pieces (outside the square). The area of the square is 60.



What is the total area of two dark pieces and one light piece together as depicted in the right-hand figure?

4. Elisa is making so-called *dubious dice*. Each face of a dubious die contains one of the numbers 1 to 6, but not all these numbers need to occur and some may occur more than once. However, from every direction it must look like a real die. This means that in each corner three different numbers must meet, no two of which add up to 7 (on a real die such pairs are always on opposite faces). For example, the numbers 1, 2, and 4 may meet in a corner, but 1, 2, and 5 may not as $2 + 5 = 7$. Of course, a normal die is an example of a dubious die as well.
Elisa is interested in the sum of the six numbers on a dubious die.
How many possible values are there for this sum?