First Round
Dutch Mathematical Olympiad

Friday, January 30, 2009

Problems

- Time available: 2 hours.
- The A-problems are multiple choice questions. Only one of the five given options is correct. Please state clearly after which letter the correct solution is stated. You get 2 points for each correct answer.
- The B-problems are open questions, which have a number (or numbers) as answer. You get 5 points for each correct answer. Please work accurately, since an error in your calculations can cause your solution to be considered wrong; then you won't get points at all for that question. Please give your answers exactly, for example $\frac{11}{81}$ or $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$.
- You are not allowed to use calculators and formula sheets; you can only use a pen, a compass and a ruler or set square. And your head, of course.
- This is a competition, not an exam. The point is that you have fun solving unusual mathematical problems. Good luck!

A-problems

A1. Ella has done three tests. During the first one, she answered 60% of the 25 questions correctly, during the second one, she answered 70% of the 30 questions correctly, and during the last one, she answered 80% of the 45 questions correctly. Now if we merge these three tests together to form one of 100 questions, what is the percentage of these 100 questions that Ella answered correctly?

(A) 68% (B) 70% (C) 72% (D) 74% (E) 76%

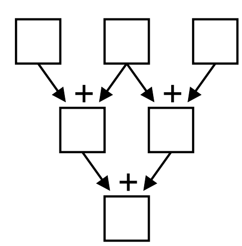
A2. How many of the integers from 10 to 99 (10 and 99 included) have the property that the sum of their digits is equal to the square of an integer? (An example: The sum of the digits of 27 is equal to $2 + 7 = 9 = 3^2$.)

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

A3. Ronald throws three dice. These dice look just like ordinary dice, but their faces are numbered differently.
The first die has the numbers 1, 1, 2, 2, 3 and 3 on it.
The second die has the numbers 2, 2, 4, 4, 6 and 6 on it.
And the third die has the numbers 1, 1, 3, 3, 5 and 5 on it.
He then adds up the three numbers he gets from rolling the three dice. What is the probability that the resulting number is odd?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

A4. Three distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are placed in the three squares in the top of the figure to the right, after which the numbers are added as described in said figure. We call Max the highest number that can appear in the bottom square, and Min the lowest number that can appear there. What is the value of Max − Min?



(A) 16 (B) 24 (C) 25 (D) 26 (E) 32

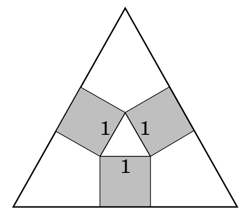
A5. The lengths of the diagonals of a rhombus have a ratio of $3 : 4$. (A rhombus is an equilateral quadrilateral.) The sum of the lengths of the diagonals is 56. What is the perimeter of this rhombus?

(A) 80 (B) 96 (C) 100 (D) 108 (E) 160

A6. Wouter is traveling by foot from his home to the fitness center. He also could have chosen to travel by bike, in which case he would travel 7 times as fast. But he left his bike at home. After walking for 1 km, continuing to walk would take just as long as walking back to get his bike, and then travel further by bike. By then, what is the distance in km to the fitness center?

(A) $\frac{8}{7}$ (B) $\frac{7}{6}$ (C) $\frac{6}{5}$ (D) $\frac{5}{4}$ (E) $\frac{4}{3}$

A7. On the sides of an equilateral triangle, we draw three squares. The sides of these squares that are parallel to the sides of the triangle are extended until they intersect. These three intersections form another equilateral triangle. Suppose that the length of a side of the original triangle is equal to 1. What is the length of a side of the large equilateral triangle?



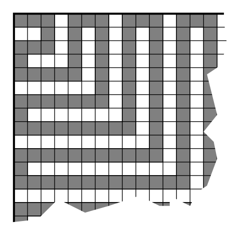
(A) $1+2\sqrt{2}$ (B) $5-\frac{1}{2}\sqrt{3}$ (C) $3\sqrt{2}$ (D) $1+2\sqrt{3}$ (E) $2\sqrt{6}$

A8. Consider all four-digit numbers where each of the digits 3, 4, 6 and 7 occurs exactly once. How many of these numbers are divisible by 44?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 12

B-vragen

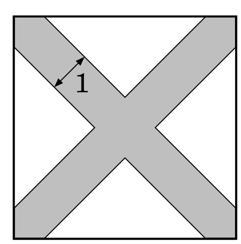
B1. On a sheet of paper, a grid of 101 by 101 white squares is drawn. A chain is formed by coloring squares grey as shown in the figure to the right. The chain starts in the upper left corner and goes on until it cannot go on any further. Only part of the grid is shown. In total, how many squares are colored grey in the original grid of 101 by 101 squares?



B2. The integer N consists of 2009 consecutive nines. A computer calculates $N^3 = (99999\ldots99999)^3$. How many nines does the number N^3 contain in total?

$$\underbrace{99999\ldots99999}_{2009\times}$$

B3. Using a wide brush, we paint the diagonals of a square tile, as in the figure. Exactly half of the area of this tile is covered with paint. Given that the width of the brush is 1, as indicated in the figure, what is the length of the side of the tile?



B4. Determine a triplet of integers (a, b, c) satisfying:

$$a+b+c=18$$
$$a^2+b^2+c^2=756$$
$$a^2=bc$$