Second round
Dutch Mathematical Olympiad

Friday 13 March 2015

- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

B-problems

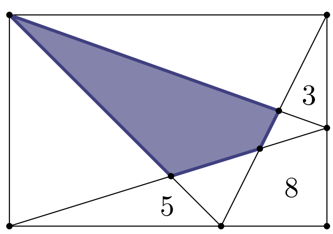
The answer to each B-problem consists of one or more numbers. A correct answer is awarded 4 points, for a wrong or incomplete answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$.

B1. We consider numbers consisting of two or more digits with no digit being 0. Such a number is called *thirteenish* if every two consecutive digits form a multiple of 13. For example: 139 is thirteenish because $13 = 1 \times 13$ and $39 = 3 \times 13$.
How many five digit numbers are thirteenish?

B2. A quadrilateral $ABCD$ has right angles at A and B. Also, $|AB| = 5$ and $|AD| = |CD| = 6$. Determine all possible values of $|BC|$.

B3. Berry has picked 756 raspberries. He divides the raspberries among himself and his friends in such a way that everyone gets the same number of raspberries. However, three of his friends are not feeling hungry and they each return a number of raspberries: exactly one quarter of their share. Berry has a healthy appetite and eats not only his own share, but the returned raspberries as well. Berry has lost count, but does know for a fact that he has eaten more than 150 raspberries.
How many raspberries did Berry eat?

B4. Four line segments divide a rectangle into eight pieces as indicated in the figure. For three of the pieces, the area is indicated as well: 3, 5, and 8.



What is the area of the coloured quadrilateral?

PLEASE CONTINUE ON THE OTHER SIDE

B5. In the cells of a 5×5-table, the numbers 1 to 5 are placed in such a way that in every row and in every column, each of the five numbers occurs exactly once. A number in a given row and column is *well-placed* if the following conditions are met.

- In that row, all smaller numbers are to the left of the number and all larger numbers are to the right of it, or conversely.
- In that column, all smaller numbers are below the number and all larger numbers are above it, or conversely.

What is the maximum number of well-placed numbers in such a table?

C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.
Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem separately).

C1. A set of different numbers are *evenly spread* if after sorting them from small to large, all pairs of consecutive numbers have the same difference. For example: 3, 11 and 7 are evenly spread, because after sorting them, both differences are 4.

a) Kees starts out with three different numbers. He adds each pair of these numbers to obtain three outcomes. According to Jan, these three outcomes can be evenly spread only if the three starting numbers were evenly spread.
Is Jan right? If so, prove this; if not, use an example to prove that Jan is wrong.

b) Jan starts out with four different numbers. He also adds each pair of them to obtain six outcomes. He wants to choose his four numbers in such a way that the six resulting numbers are evenly spread.
Is this possible? If so, give an example; if not, prove that it is impossible.

C2. We consider rectangular boards consisting of $m \times n$ cells that are arranged in m (horizontal) rows and n (vertical) columns. We want to colour each cell of the board black or white in such a way that the following rules are obeyed.

- For every row, the number of white cells equals the number of black cells.
- If a row and a column meet in a *black* cell, the row and column contain equal numbers of black cells.
- If a row and a column meet in a *white* cell, the row and column contain equal numbers of white cells.

Determine all pairs (m, n) for which such a colouring is possible.