

Second round

Dutch Mathematical Olympiad



Friday 23 March 2012

- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

B-problems

The answer to each B-problem is a number. A correct answer is awarded 4 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer. NOTE: all answers should be given in exact form, like $\frac{11}{81}$ or 5^8 or $\frac{1}{4}(\sqrt{5} + \pi)$.

- B1.** In this addition, each letter represents a digit (0 to 9). Different letters represent different digits. Determine the value of $W \times R$.

$$\begin{array}{r} T W E E D E \\ R O N D E + \\ \hline 2 3 0 3 1 2 \end{array}$$

- B2.** All 2012 camels in the Netherlands are to be distributed among 40 pastures. No two pastures are allowed to get the same number of camels. The pasture in the city centre of Amsterdam has to get the largest number of camels.

At least how many camels have to be placed in that pasture?

- B3.** One of the four dwarfs Anne, Bert, Chris and Dirk stole the king's gold. The dwarfs, who know each other very well, each make two statements. If a dwarf is a liar, then at least one of these two statements is a lie. If the dwarf is not a liar, then both statements are true.

Anne says: "Bert is a liar." and "Chris or Dirk stole the gold."

Bert says: "Chris is a liar." and "Dirk or Anne stole the gold."

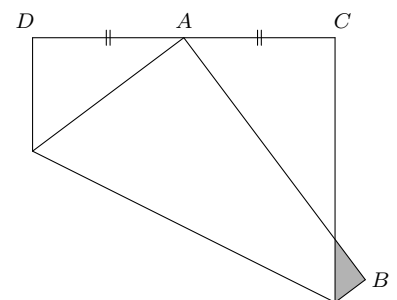
Chris says: "Dirk is a liar." and "Anne or Bert stole the gold."

Dirk says: "Anne is a liar." and "Bert or Chris stole the gold."

How many of these eight statements are true?

- B4.** On each of the 10,000 squares of a 100×100 -chess board a number is written. Along the top row the numbers 0 to 99 are written from left to right. In the left column the numbers 0 to 99 are written from top to bottom. The sum of the four numbers in a 2×2 -block always equals 20. Which number is written in the bottom right square of the board?

- B5.** A square $ABCD$ with side length 8 is folded in such a way that vertex A becomes the midpoint of CD (see figure). Find the area of the grey triangle.



C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.

Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem separately).

C1. You have one card with the number 12 on it. You are allowed to add new cards to your collection according to the following rules.

- If you have a card with the number a on it, then you are allowed to make a new card with the number $2a + 1$ on it.
- If you have a card with the number b on it and b is divisible by 3, then you are allowed to make a new card with the number $\frac{b}{3}$ on it.

- (a) Show that you can make a card with the number 29 on it.
- (b) Show that you can make make a card with the number $2^{2012} - 1$ on it.
- (c) Show that you can never make a card with the number 100 on it.

C2. Given is a triangle ABC , a point D on line segment AC and a point E on line segment AB . The intersection of BD and CE is called S . The midpoint of line segment CS is called M . The line BM intersects line segment CD in point T . Finally we are given that $|BE| = |ES| = 1$ and $|CD| = |DS| = 2$.

Prove that $|AB| = |AT|$.

You have to write your argument step-by-step in text and formulas. Elements in your drawing that are not supported by written arguments will not be awarded any points.

