

# Second Round

## Dutch Mathematical Olympiad



Friday 25 March 2011

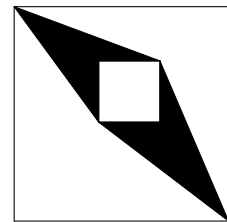
- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

### B-problems

The answer to each B-problem is a number. A correct answer is awarded 4 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer. NOTE: all answers should be given in exact form, like  $\frac{11}{81}$  or  $5^8$  or  $\frac{1}{4}(\sqrt{5} + \pi)$ .

- B1.** At a gala, a number of pairs (consisting of one man and one woman) are dancing, in such a way that  $\frac{2}{3}$  of the women present is dancing with  $\frac{3}{5}$  of the men present. What part of those present at the gala is dancing?

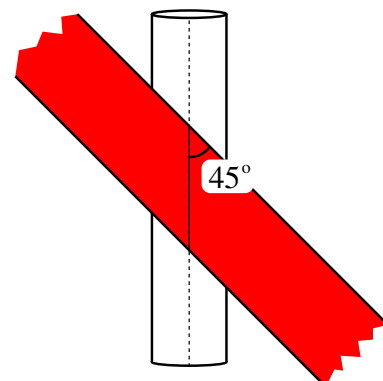
- B2.** A square with edges of length 2 is inside of a square with edges of length 7. The edges of the smaller square are parallel to the ones of the larger one. What is the area of the black-coloured part?



- B3.** In a classroom, there are 23 students who chose to learn a foreign language, namely German and French. Of those 23 students, 10 are girls, and 11 of those 23 students have chosen French as their foreign language. The number of girls that have chosen French, plus the number of boys that have chosen German, is equal to 16. What is the number of girls that have chosen French?

- B4.** We have a deck of 10.000 cards, numbered from 1 to 10.000. A step consists of removing all the cards that has a square on it, and then renumbering the remaining cards, starting from 1, in a successive way. What is the number of steps needed to remove all but one card?

- B5.** We put a red ribbon over a cylindrical white pole, under an angle of 45 degrees. The ribbon is then wound tightly around the pole (without creases). In this way, we get a red spiral around the pole. Between the red spiral, a white spiral runs around the pole; that is the part of the pole that is not covered by the ribbon. The radius of the pole is 2 cm. It turns out that the white and red spirals have the same widths. What is the width of the ribbon?



PLEASE CONTINUE ON THE OTHER SIDE

## C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.

Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem separately).

**C1.** Determine all triples  $(a, b, c)$  of positive integers with the following properties:

- we have  $a < b < c$ , and  $a$ ,  $b$  and  $c$  are three successive odd integers;
- the number  $a^2 + b^2 + c^2$  consists of four equal digits.

**C2.** Thirty students participate in a mathematical competition with sixteen questions. They have to answer each question with a number. If a student answers a question correctly within a minute, he gets 10 points for that question. If a student answers a question correctly, but not within one minute, then he gets 5 points for that question. And if a student answers a question incorrectly, he gets no points at all for that question.

After the competition has ended, it turns out that from all the 480 answers that were given, more than half was correct and given within a minute. The number of answers that were correct, but not given within a minute, turns out to be equal to the number of incorrect answers.

Show that there are two students with the same total score.