Final round
Dutch Mathematical Olympiad

Friday 14 September 2012
Eindhoven University of Technology

- Available time: 3 hours.
- Each problem is worth 10 points. A description of your solution method and clear argumentation are just as important as the final answer.
- Calculators and formula sheets are not allowed. You can only bring a pen, ruler (set square), compass and your math skills.
- Use a separate sheet for each problem. Good luck!

1. Let a, b, c, and d be four distinct integers.
Prove that $(a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$ is divisible by 12.

2. We number the columns of an $n \times n$-board from 1 to n. In each cell, we place a number. This is done in such a way that each row precisely contains the numbers 1 to n (in some order), and also each column contains the numbers 1 to n (in some order). Next, each cell that contains a number greater than the cell's column number, is coloured blue. In the figure below you can see an example for the case $n = 3$.

1	2	3
3	1	2
1	2	3
2	3	1

(a) Suppose that $n = 5$. Can the numbers be placed in such a way that each row contains the same number of blue cells?

(b) Suppose that $n = 10$. Can the numbers be placed in such a way that each row contains the same number of blue cells?

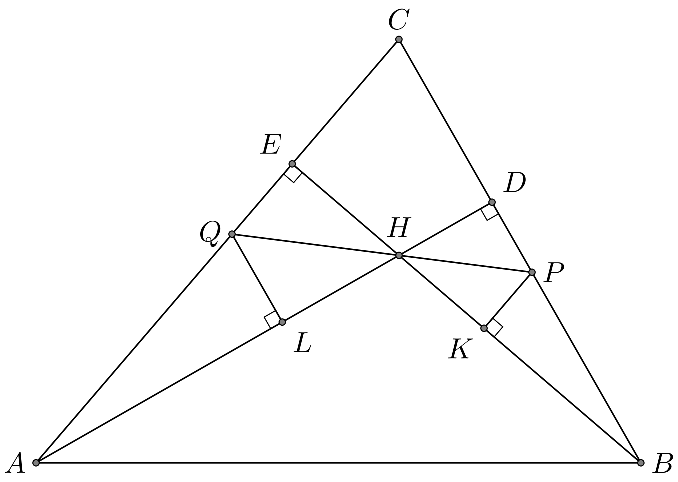
3. Determine all pairs (p, m) consisting of a prime number p and a positive integer m, for which

$$p^3 + m(p+2) = m^2 + p + 1$$

holds.

PLEASE CONTINUE ON THE OTHER SIDE

4. We are given an acute triangle ABC and points D on BC and E on AC such that AD is perpendicular to BC and BE is perpendicular to AC. The intersection of AD and BE is called H. A line through H intersects line segment BC in P, and intersects line segment AC in Q. Furthermore, K is a point on BE such that PK is perpendicular to BE, and L is a point on AD such that QL is perpendicular to AD.



Prove that DK and EL are parallel.

5. The numbers 1 to 12 are arranged in a sequence. The number of ways this can be done equals $12 \times 11 \times 10 \times \cdots \times 1$. We impose the condition that in the sequence there should be exactly one number that is smaller than the number directly preceding it.
How many of the $12 \times 11 \times 10 \times \cdots \times 1$ sequences meet this demand?