Junior Wiskunde Olympiade
Problems part 2

Saturday 29 September 2018
Vrije Universiteit Amsterdam

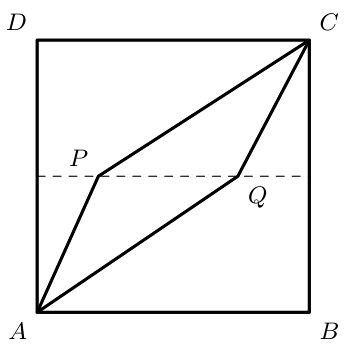
- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct and complete answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**

1. Anne, Bert, Christiaan, Dirk, and Eveline are particpating in a chess tournament. They find out that their average age is exactly 28 years. Exactly one year later Anne, Bert, Christiaan, and Dirk participate together with Freek in the tournament. This time, their average age is exactly 30 years.
How many years older is Freek compared to Eveline?

2. What is the smallest positive integer x for which the outcome of $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}+\frac{x}{5}+\frac{x}{6}$ is an integer?

3. For each of the two fractions $\frac{2018}{2011}$ and $\frac{2054}{2019}$ we subtract the same integer a from both the numerator and the denominator. The two fractions we get, are equal.
What integer is a?

4. The sides of the square $ABCD$ have length 10. The points P and Q lie on the line connecting the midpoints of AD and BC. If we connect P with A and C, and also Q with A and C, then the square is divided into three parts having equal area.
What is the length of PQ? (*The figure is not drawn to scale.*)



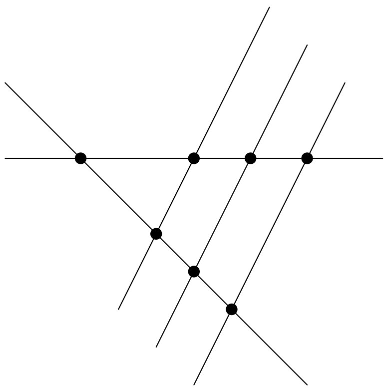
PLEASE CONTINUE ON THE OTHER SIDE

5. Jan has written the number A in two different ways as a fraction. Unfortunately, two numbers have become invisible due to spilled ink spots:

$$A = \frac{\blacksquare + 3}{12} = \frac{15}{26 - \blacksquare}.$$

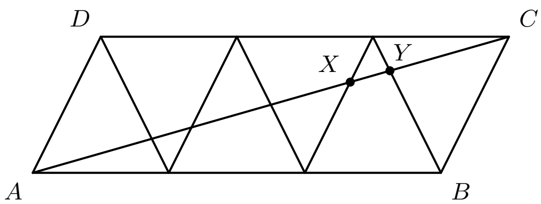
Lia has found Jan's note and wants to find all possibilities for the number A. She knows that the numbers underneath the ink spots must be positive integers, but the number A does not have to be an integer. Because she does not know which numbers are hidden below the ink spots, she searches for all combinations for which the equality holds. In this way, she finds multiple possible values for A. The largest one she calls $A_{\max}$ and the smallest one she calls $A_{\min}$. Determine $A_{\max} - A_{\min}$.

6. Tim draws five straight lines in a plane. The lines are continuing indefinitely, and there are no three lines going through the same point. For each intersection point of two lines he gets one sweet, and for each set of two or more parallel lines he gets one sweet as well. For instance, in the example below, he gets 8 sweets because firstly there are 7 intersection points, and secondly there is 1 set of three parallel lines, which is worth another sweet.



What are all possible numbers of sweets that Tim can get?

7. Six equilateral and equally sized triangles are glued to form a parallelogram $ABCD$, see the figure.



The length of AC is 10. What is the length of XY?
Attention: the figure is not drawn to scale.

8. In a class room there are a number of students. They find out that for each triplet of students, the following two statements are both true:

- Two of them never wrote a report together.
- Two of them did once write a report together.

What is the maximum possible number of students in the class room?