

# Junior Wiskunde Olympiade

## Problems part 2



Saturday 1 October 2016  
Vrije Universiteit Amsterdam

- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**

1. All vehicle registration plate numbers in the country Wissewis consist of three two-digit numbers. A plate number is considered beautiful if it has the following two properties:

- it consists of six distinct digits;
- the first number is smaller than the second number and the second number is smaller than the third number.

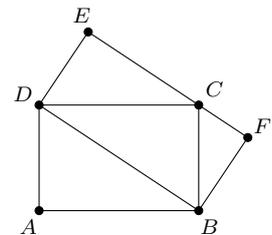
An example of a beautiful plate number is 03-29-64.

How many beautiful plate numbers are there that have 61 as the first number?

2. Alice, Bob, Carla, Daan, and Eva are standing in this order along a circle (Bob is standing to the left of Alice). Each of them has a number of sweets, they have 100 sweets in total. All at the same time, they give part of their sweets to their left neighbour: Alice gives away  $\frac{1}{3}$  of her sweets, Bob  $\frac{1}{4}$ , Carla  $\frac{1}{5}$ , Daan  $\frac{1}{6}$ , and Eva  $\frac{1}{7}$ . After this, everybody has the same number of sweets as before.

How many sweets does Eva have?

3. In the figure on the right, rectangles  $ABCD$  and  $BDEF$  are shown. The length of  $AB$  is 8 and the length of  $BC$  is 5. What is the area of pentagon  $ABFED$ ?

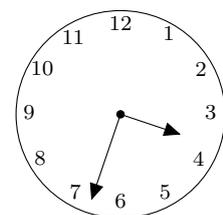


4. In this problem we consider three-digit numbers of which no digit is a zero. Such a number is called a *lucky number* if:

- the number is divisible by 4, and
- if you change the order of the three digits, you will still always get a number divisible by 4.

For example, the number 132 is not a lucky number, because 132 is divisible by 4, but 231 is not. How many lucky numbers are there?

5. How many times a day (which is 24 hours) are the small hand and the big hand of the clock perpendicular?



6. Janneke, Karin, Lies, Marieke, and Nadine participated in a running race. They all finished at distinct times except for two of them; they finished at the same time. Moreover, we know that:

- at least three runners finished before Janneke;
- after Karin finished but before Lies finished, exactly two others crossed the finish line;
- Marieke was not the first to finish;
- shortly after Nadine finished, Janneke crossed the finish line; nobody else was in-between.

Which two runners finished at the same time?

7. For all positive integers  $a$  and  $b$  we make the number  $a \heartsuit b$ . The following rules hold:

- rule 1:  $1 \heartsuit 1 = 1$ ;
- rule 2:  $a \heartsuit b = b \heartsuit a$ ;
- rule 3:  $a \heartsuit (b + c) = a + (a \heartsuit b) + (a \heartsuit c)$ .

From these rules it follows, for example, that

$$2 \heartsuit 1 = 1 \heartsuit 2 = 1 \heartsuit (1 + 1) = 1 + 1 \heartsuit 1 + 1 \heartsuit 1 = 1 + 1 + 1 = 3.$$

Calculate  $20 \heartsuit 16$ .

8. We create a sequence of numbers. To get the next number in the sequence, we repeatedly do the following:

- if the previous number is odd: multiply this number by itself and add 3;
- if the previous number is even: divide this number by 2.

For example, when we start with 5, we obtain  $5 \times 5 + 3 = 28$  as second number and  $\frac{28}{2} = 14$  as third number in the sequence. As starting number we are allowed to choose any of the numbers from 1 to 1000.

For how many of these starting numbers will the tenth number in the sequence be smaller than 10?