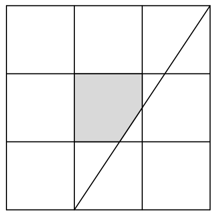
Junior Wiskunde Olympiade
Problems part 2

Saturday 3 October 2015
Vrije Universiteit Amsterdam

- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**



1. In the figure on the right there is a square with side length 3. The square is divided into nine equal squares. Then, another line is drawn that creates a pentagon inside the middle square (coloured grey).
What is the area of this pentagon?

2. A palindromic number is a number that is the same when read from left to right as when read from right to left. A number does not start with the digit 0. To a six-digit palindromic number the palindromic number 21312 is added. The result is a seven-digit palindromic number.
What is this resulting number?

3. Using exactly six zeros and six ones we create two or more numbers which we then multiply. For instance, we could get $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$, or $10{,}011 \times 100 \times 1{,}100 = 1{,}101{,}210{,}000$.
What is the largest possible result we could get in this way?

4. Pieter is staying at a hotel. The hotel has a ground floor (numbered 0) and seven additional floors numbered 1 to 7. Pieter wants to make a trip by elevator. He starts on one of the floors 1 to 7 and ends at the ground floor. In between, he tavels from floor to floor, never stopping at a previously visited floor and never stopping at the ground floor (except for the last stop).

The distance between any two consecutive floors is 3 metres. If, for example, Pieter starts at floor 3, then goes to floor 6, then to floor 4 and finally to the ground floor, he travels a total of $(3 + 2 + 4) \times 3 = 27$ metres.
What is the maximum length of his trip in metres?

5. How many 3-digit numbers (the first digit cannot be 0) have the property that adding all the digits gives a strictly greater result than multiplying all the digits?

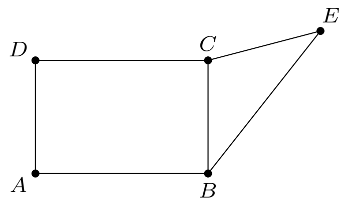
PLEASE CONTINUE ON THE OTHER SIDE

6. The mayor of a town wants to build a network of express trams. She wants it to meet the following conditions:

- There are at least two distinct tram lines.
- Each tram line serves exactly three stops (also counting the start and terminus).
- For each two tram stops in the town there is exactly one tram line that serves both stops.

What is the minimum number of stops that the mayor's tram network can have?

7. A rectangle $ABCD$ and a point E are given. Line segmnents BE and BA have the same length. Line segments CE and CB also have the same length. Moreover, the area of rectangle $ABCD$ is four times as large as the area of triangle BCE. Side AD has length 10.
What is the length of diagonal AC?



8. We consider ways to fill a 5×5-board by writing a 1 or a 3 in each square. Such a filling is called *balanced* if the following holds:

- If you take an arbitrary 3×3-square of the board and multiply all the numbers that it contains, and after that you do the same for an arbitrary 4×4-square, then the second result is always three times as large as the first result.

In the figure on the right, you see a filling that is *not* balanced. For example, when multiplying the numbers in the indicated 4×4-square, the result is *nine times* as large as the result obtained by multiplying the numbers in the indicated 3×3-square.

Give (on the answer form) a balanced filling of the board containing a maximum number of 3-s.

1	1	3	3	1
3	1	1	1	3
1	3	1	3	1
3	1	1	3	1
1	1	3	1	1