Second round
Dutch Mathematical Olympiad

Friday 16 March 2018

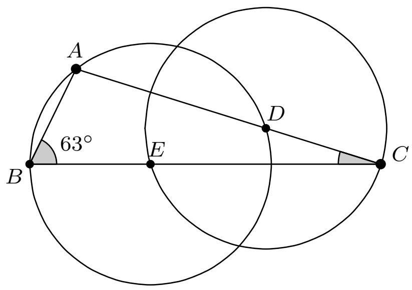
- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler, set square, and of course your mental skills.
- Good luck!

B-problems

For the B-problems only the answer has to be handed in (for example, a number). No explanation is required. A correct answer is awarded 4 points, for a wrong or incomplete answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact and reduced form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi+1$, or 3^{100}.

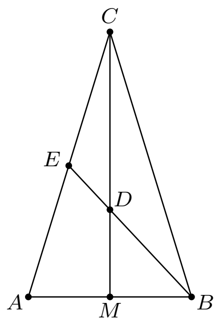
B1. Anouk, Bart, Celine, and Daan have participated in a math competition. Each of their scores is a positive integer. The sum of the scores of Bart and Daan is the same as the sum of the scores of Anouk and Celine. The sum of the scores of Anouk and Bart is higher than the sum of the scores of Celine and Daan. Daan's score is higher than the sum of the scores of Bart and Celine. Write down the names of the four students in decreasing order of their scores.

B2. In the figure, you can see a triangle ABC. The angle at B is equal to 63 degrees. Side AC contains a point D, and side BC contains a point E. Points B, A, and D lie on a circle with centre E. Points E and C lie on a circle with centre D.
What is the angle at point C?
(Warning: the figure is not drawn to scale.)



B3. A *palindromic number* is a positive integer (consisting of one or more digits) that remains the same when the digits are reversed. For example: 1245421 and 333 are palindromic numbers, but 345 and 100 are not. There is exactly one palindromic number n with the following property: if you subtract 2018 from n, the result is again a palindromic number.
What number is n?

B4. Triangle ABC is isosceles with apex C. The midpoint of AB is point M. On segment CM there is a point D such that $\frac{|CD|}{|DM|}=\frac{3}{2}$. Line BD intersects segment AC in point E.
Determine $\frac{|CE|}{|EA|}$. *(Warning: the figure is not drawn to scale.)*



B5. A *sawtooth number* is a positive integer with the following property: for any three adjacent digits, the one in the middle is either greater than its two neighbours or smaller than its two neighbours. For example, the numbers 352723 and 314 are sawtooth numbers, but 3422 and 1243 are not.
How many 8-digit sawtooth numbers exist, for which each of the digits is equal to 1, 2, or 3?

PLEASE CONTINUE ON THE OTHER SIDE

C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning that shows correctness of your answer. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.
Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem separately).

C1. You have n balls that are numbered from 1 to n. You need to distribute the balls over two boxes. The *value* of a box is the sum of the numbers of the balls in that box. Your distribution of balls must obey the following rules:

- Each box has at least one ball.
- The two boxes do not have the same number of balls.
- The value of the box with the least number of balls must be at least 2 more than the value of the box with the most balls.

Determine for which positive integers n this is possible.
(Prove that for those values of n it is indeed possible, and prove that it is not possible for other values of n.)

C2. In this problem we consider squares: numbers of the form m^2 where m is an integer.

(a) Does there exist an integer a such that $16 + a$, $3 + a$, and $16 \cdot 3 + a$ are squares?
If so, give such a number a and show that the three numbers are indeed squares.
If not, prove that such a number a does not exist.

(b) Does there exist an integer a such that $20 + a$, $18 + a$, and $20 \cdot 18 + a$ are squares?
If so, give such a number a and show that the three numbers are indeed squares.
If not, prove that such a number a does not exist.

(c) Prove that for every odd integer n there exists an integer a such that $2018 + a$, $n + a$, and $2018 \cdot n + a$ are squares.