Second round
Dutch Mathematical Olympiad

Friday 11 March 2016

- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

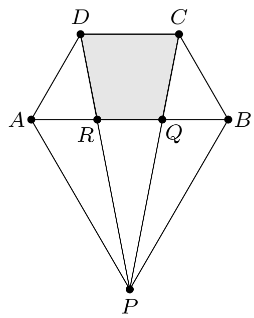
B-problems

For the B-problems only the answer has to be handed in (for example, a number). No explanation is required. A correct answer is awarded 4 points, for a wrong or incomplete answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi+1$.

B1. How many of the integers from 10 to 99 have the property that the number equals four times the sum of its two digits?

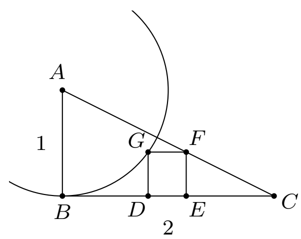
B2. In a box there are 100 cards that are numbered from 1 to 100. The numbers are written on the cards. While being blindfolded, Lisa is going to draw one or more cards from the box. After that, she will multiply together the numbers on these cards.
Lisa wants the outcome of the multiplication to be divisible by 6. How many cards does she need to draw to make sure that this will happen?

B3. In the trapezium $ABCD$ the sides AB and CD are parallel and we have $|BC| = |CD| = |DA| = \frac{1}{2}|AB|$. On the exterior of side AB there is an equilateral triangle BAP. The point Q is the intersection of PC and AB, and R is the intersection of PD and AB (see the figure). The area of triangle BAP is 12.
Determine the area of quadrilateral $QCDR$.



B4. At championships of 'The Settlers of Catan', three participants play against each other in each game. At a certain championship, three of the participants were girls and they played against each other in the first game. Each pair of participants met each other in exactly one game and in each game at least one girl was playing.
What is the maximum number of participants that could have competed in the championship?

B5. Triangle ABC has a right angle at B. Moreover, the side length of AB is 1 and the side length of BC is 2. On the side BC there are two points D and E such that E lies between C and D and $DEFG$ is a square, where F lies on AC and G lies on the circle through B with centre A.
Determine the length of DE.



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C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.
Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem separately).

C1. A positive integer is called 2016-*invariant* if the sum of its digits does not change when you add 2016 to the integer. For example, the integer 8312 is 2016-invariant: the sum of the digits of 8312 is $8 + 3 + 1 + 2 = 14$, and this equals the sum of the digits of $8312 + 2016 = 10328$, which is $1 + 0 + 3 + 2 + 8 = 14$.

(a) Determine the largest four-digit number that is 2016-invariant.

(b) There are 9999 positive integers having at most four digits.
Determine how many of these are 2016-invariant.

C2. For the upcoming exam, the desks in a hall are arranged in n rows containing m desks each. We know that $m \geqslant 3$ and $n \geqslant 3$. Each of the desks is occupied by a student. Students who are seated directly next to each other, in front of each other, or diagonally from each other, are called *neighbours*. Thus, students in the middle of the hall have 8 neighbours. Before the start of the exam, each student shakes hands once with each of their neighbours. In total, there are 1020 handshakes.
Determine the number of students.

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