First round
Dutch Mathematical Olympiad

21 January – 31 January 2019

- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact and simplified form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi+1$, or 3^{100}.
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 1 February on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

1. Arthur has written down five distinct positive integers smaller than 10. If you add any two of these five numbers, then the result will never be equal to 10.
Which number did Arthur write down for sure?

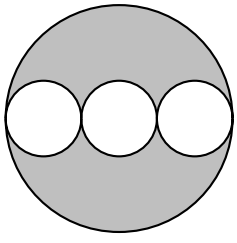
A) 1 B) 2 C) 3 D) 4 E) 5

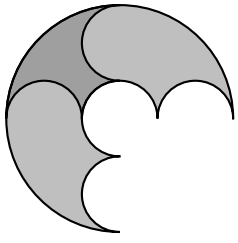
2. On a 2019×2019 chess board, there is a contagious disease. Each day some of the squares on the chess board are sick and the rest are healthy. A healthy square bordering a sick square (along a side), becomes sick itself the next day. A sick square will always be healthy the next day. A healthy square that has been sick before, can become sick again (if it is infected by one of the adjacent squares). On day 1, only the middle square is sick.
How many squares are sick on day 100?

A) 200 B) 298 C) 396 D) 9999 E) 10000

3. Out of a circular disk of radius 3, we cut three small disks of radius 1 in the way depicted in the figure on the left. This causes the remainder of the big disk to fall apart into two pieces. The bottom part is rotated 90 degrees and is put on top of the upper part as shown in the figure on the right. The part where the two pieces overlap is coloured a bit darker.
What is the total area of the figure on the right (i.e. both the light and dark grey parts together)?

A) 4π B) $\frac{9}{2}\pi$ C) $\frac{19}{4}\pi$ D) 5π E) $\frac{21}{4}\pi$





PLEASE CONTINUE ON THE OTHER SIDE

4. There are 13 distinct multiples of 7 that consist of two digits. You want to create a longest possible chain consisting of these multiples, where two multiples can only be adjacent if the last digit of the left multiple equals the first digit of the right multiple. You can use each multiple at most once. For example, 21 – 14 – 49 is an admissible chain of length 3.
What is the maximum length of an admissible chain?

A) 6 B) 7 C) 8 D) 9 E) 10

5. In a table with two rows and five columns, each of the squares is coloured black or white according to the following rules:

- Two adjacent columns may never have the same number of black squares.
- Two 2×2-squares that overlap in one column may never have the same number of black squares.

How many possible colourings of the table comply with these rules?

A) 6 B) 8 C) 12 D) 20 E) 24

6. Which of the following numbers is the largest number you can get by separating the numbers 1, 2, 3, 4, and 5 by using each of the operations $+$, $-$, $:$, and $\times$ exactly once, where you may use parentheses to indicate the order in which the operations should be executed? For example: $(5-3)\times(4+1):2=5$.

A) 21 B) $\frac{53}{2}$ C) 33 D) $\frac{69}{2}$ E) 35

7. Agatha, Isa and Nick each have a different kind of bike. One of them has an electric bike, one has a racing bike, and one has a mountain bike. The bikes have different colours: green, blue and black. The three owners make two statements each, of which one is true and the other is false:

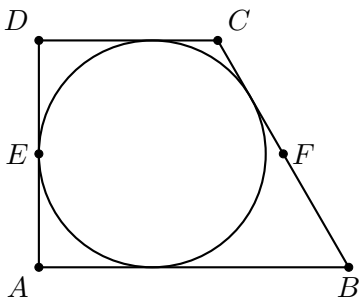
- Agatha says: "I have an electric bike. Isa has a blue bike."
- Isa says: "I have a mountain bike. Nick has an electric bike."
- Nick says: "I have a blue bike. The racing bike is black."

Exactly one of the following statements is certainly true. Which one?

A) Agatha has a green bike.
B) Agatha has a mountain bike.
C) Isa has a green bike.
D) Isa has a mountain bike.
E) Nick has an electric bike.

8. Quadrilateral $ABCD$ has right angles at A and D. A circle of radius 10 fits neatly inside the quadrilateral and touches all four sides. The length of edge BC is 24. The midpoint of edge AD is called E and the midpoint of edge BC is called F.
What is the length of EF?

A) $\frac{43}{2}$ B) $\frac{13}{2}\sqrt{11}$ C) $\frac{33}{5}\sqrt{11}$ D) 22 E) $\frac{45}{2}$

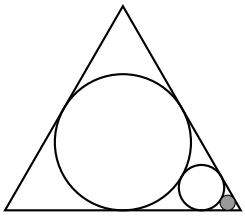


B-problems

1. Every day, Maurits bikes to school. He can choose between two different routes. Route B is 1.5 km longer than route A. However, because he encounters fewer traffic lights, his average speed along route B is 2 km/h higher than along route A. This makes that travelling along the two routes takes exactly the same amount of time.
How long does it take for Maurits to bike to school?

2. Starting with a positive integer, a *fragment* of that number is any positive number obtained by removing one or more digits from the beginning and/or end of that number. For example: the numbers 2, 1, 9, 20, 19, and 201 are the fragments of 2019.
What is the smallest positive integer n such that the following holds: there is a fragment of n such that when you add this fragment to n itself, you get 2019?

3. Inside an equilateral triangle, a circle is drawn that touches all three sides. The radius of the circle is 10. A second, smaller, circle touches the first circle and two sides of the triangle. A third, even smaller, circle touches the second circle and two sides of the triangle (see the figure).
What is the radius of the third circle?



4. Alice has a number of cards. Each card contains three of the letters A to I. For any choice of two of those letters, there is at least one card that contains both letters.
What is the smallest number of cards that Alice can have?