First round
Dutch Mathematical Olympiad

18 January – 28 January 2016

- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$.
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 29 January on the website: www.wiskundeolympiade.nl.
- Good luck!

A-problems

1. Frank has two integers that add up to 26. Kees adds two more integers to it and gets 41. Pieter adds another two integers and gets 58.
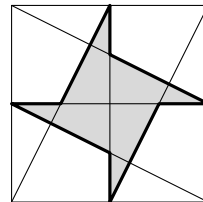   At least how many of the six integers that were added up are *even*?

   A) 0   B) 1   C) 2   D) 3   E) 4

2. In a square with side length 12, line segments are drawn between the vertices and the midpoints of the sides and between the midpoints of opposite sides (see the figure). In this way, a star shaped figure is created.
   What is the area of this figure?

   A) 12   B) 16   C) 20   D) 36   E) 48

3. A positive integer is called *fully divisible* if it is divisible by each of its digits. Moreover, these digits must all be distinct (and nonzero). For example, 162 is fully divisible, because it is divisible by 1, 6, and 2.
   How many fully divisible two-digit integers are there?

   A) 4   B) 5   C) 6   D) 7   E) 8

4. An *eight* is a figure consisting of two equal circles touching each other, like 8, ∞ or ℰ. In the figure you see 66 circles stacked in the shape of a triangle.
   How many eights can you find in this stack?

   A) 99   B) 108   C) 120   D) 135   E) 165
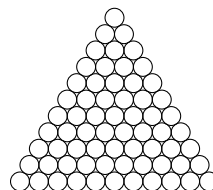
5. Five integers are written around a circle. Two neighbouring numbers never add up to a multiple of three. Also, a number and its two neighbours never add up to a multiple of three.
   How many of the five integers are multiples of three?

   A) 1   B) 2   C) 3   D) 4   E) 2 and 3 are both possible

6. In the figure you see a wire-frame model of a $2\times2\times2$-cube consisting of 8 small cubes with side length 1 dm. This figure uses 54 dm of wire.
How many dm of wire are needed for a wire-frame model of a $10\times10\times10$-cube consisting of one thousand small cubes with side length 1 dm?

A) 121   B) 1000   C) 1210   D) 3000   E) 3630
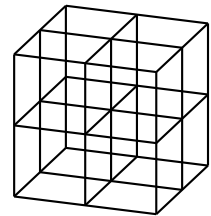
7. A square board is divided into 4×4 squares. At the start, all squares are white. Now, we want to colour some of the squares blue, in such a way that each blue square will be adjacent to exactly one white square (two squares are called adjacent if they have a side in common).
What is the maximum number of squares that we can colour blue?

A) 6   B) 8   C) 10   D) 12   E) 14

8. For three *distinct* positive integers a, b, and c we have $a + 2b + 3c < 12$.
Which of the following inequalities is certainly satisfied?

A) $3a + 2b + c < 17$ B) $a + b + c < 7$ C) $a - b + c < 4$
D) $b + c - a < 3$ E) $3b + 3c - a < 6$

B-problems

1. We construct a list of all positive integers that divide 707070. The numbers are listed in decreasing order. The first number in the list is therefore 707070 and the last one is 1.
What is the seventh number in the list?

2. In the AO-language all words consist of only A's and O's and every possible sequence of A's and O's is a word. There are, for example, eight three letter words: 'OOO', 'OOA', 'OAO',..., 'AAO', and 'AAA'. Words that contain the letter combinations 'AO' and 'OA' equally often are called *special*. For example, 'AOAAOOOAA' is special, because the word contains both letter combinations 'AO' and 'OA' twice.
Find a special word consisting of four A's and four O's with the additional property that after removing any of its letters, the resulting seven letter word is again special.

3. In the square $ABCD$ lies a point U such that BU and AB have the same length. Point V is the intersection of BU and the diagonal AC. The size of angle DAU is 28 degrees.
What is the size of the angle at V in triangle BVC?
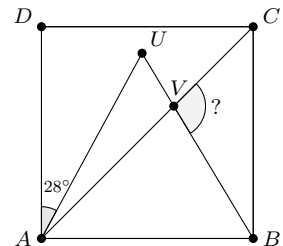
4. Seven people are suspects of a theft:
Alex, a brown-haired man with blue eyes;   **Eva**, a brown-haired woman with blue eyes;
Boris, a blond man with green eyes;   **Felix**, a brown-haired man with brown eyes;
Chris, a blond man with brown eyes;   **Gaby**, a blond woman with blue eyes.
Denise, a blond woman with brown eyes;

Detectives Helga, Ingrid, and Julius know that one of the suspects is the thief. After conducting some investigations they share their information.

> Helga: "I know the eye and hair colour of the thief, but I do not know who the thief is."
> Ingrid did not hear Helga and says:
>> "I know the hair colour and the gender, but I do not know who the thief is."
> At last, Julius says:
>> "First I knew only the gender of the thief, but after hearing you I know who the thief is."

The detectives spoke the truth. Who is the thief?