Final round
Dutch Mathematical Olympiad

Friday 13 September 2013
Eindhoven University of Technology

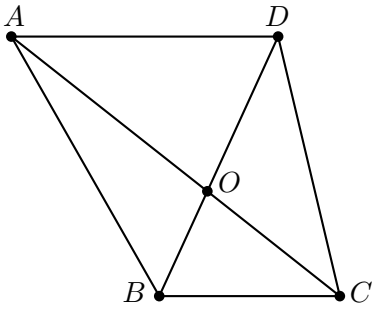
- Available time: 3 hours.
- Each problem is worth 10 points. Points can also be awarded to partial solutions.
- Write down all the steps of your argumentation. A clear reasoning is just as important as the final answer.
- Calculators and formula sheets are not allowed. You can only bring a pen, ruler (set square), compass and your math skills.
- Use a separate sheet for each problem and also hand in your draft sheets (for each problem separately!). Good luck!

1. In a table consisting of n by n small squares some squares are coloured black and the other squares are coloured white. For each pair of columns and each pair of rows the four squares on the intersections of these rows and columns must not all be of the same colour.
What is the largest possible value of n?

2. Find all triples (x, y, z) of real numbers satisfying

$$x + y - z = -1, \qquad x^2 - y^2 + z^2 = 1 \qquad \text{and} \qquad -x^3 + y^3 + z^3 = -1.$$

3. The sides BC and AD of a quadrilateral $ABCD$ are parallel and the diagonals intersect in O. For this quadrilateral $|CD| = |AO|$ and $|BC| = |OD|$ hold. Furthermore CA is the angular bisector of angle BCD.
Determine the size of angle ABC.



Attention: the figure is not drawn to scale.
You have to write down your reasoning step by step in text and formulas. No points will be awarded for annotations in a picture alone.

4. For a positive integer n the number $P(n)$ is the product of the positive divisors of n. For example, $P(20) = 8000$, as the positive divisors of 20 are 1, 2, 4, 5, 10 and 20, whose product is $1 \cdot 2 \cdot 4 \cdot 5 \cdot 10 \cdot 20 = 8000$.

(a) Find all positive integers n satisfying $P(n) = 15n$.

(b) Show that there exists no positive integer n such that $P(n) = 15n^2$.

5. The number S is the result of the following sum:

$$1 + 10 + 19 + 28 + 37 + \cdots + 10^{2013}.$$

If one writes down the number S, how often does the digit '5' occur in the result?