

First round

Dutch Mathematical Olympiad



Friday 27 January 2012

- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- The answer to each B-problem is a number or multiple numbers. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$.
- Formula sheets, scissors and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

A-problems

A1. In the multiplication table on the right, the stars represent positive integer numbers such that the table is correct.

×	*	*	*	7
*	24	*	*	56
*	*	36	8	*
*	*	27	6	*
6	18	*	*	42

What is the largest number that occurs more than once among the entries of the full 5×5 -table?

- A) 6 B) 8 C) 9 D) 12 E) 18

A2. A palindromic number is a number that does not change when the order of its digits is reversed, like 707 and 154451. Leon lists all 5-digit palindromic numbers (numbers do not start with the digit 0) in increasing order.

Which number is 12th on Leon's list?

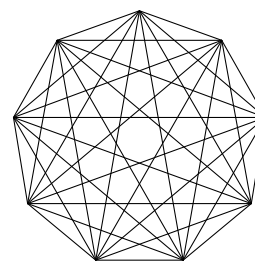
- A) 11111 B) 11211 C) 12221 D) 12321 E) 12421

A3. The figure shows a regular nonagon along with all its diagonals. Consider all triangles whose three different vertices are vertices of the nonagon.

How many of these triangles are isosceles?

(An isosceles triangle is a triangle having two or three equal sides.)

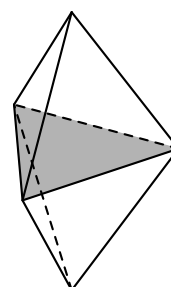
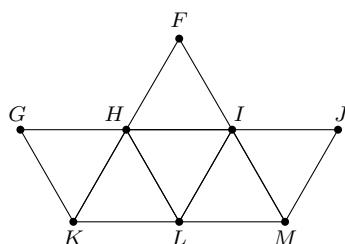
- A) 27 B) 30 C) 33 D) 36 E) 39



A4. Below is a paper model of a *dipiramid*. The unfolded version is depicted on the left, and the resulting folded dipiramid is depicted on the right.

Which three points of the unfolded model correspond to vertices of the gray triangle?

- A) F, G and L B) H, I and M C) G, H and I
 D) H, K and L E) F, I and K



PLEASE CONTINUE ON THE OTHER SIDE

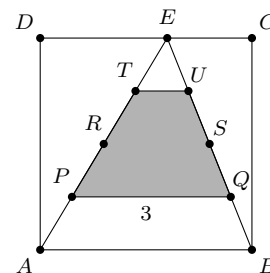
- A5.** Frank has a drawer containing single socks. There are 10 red socks, the other socks are blue. Without looking, he picks a number of socks from the drawer. The number of socks he needs to pick to be sure of getting at least two red socks, is twice the number he needs to pick for getting at least two blue socks.

How many socks does the drawer contain?

- A) 14 B) 18 C) 26 D) 32 E) 40

- A6.** Point E lies on side CD of a square $ABCD$. Line segment AE is divided into four equal parts by points P , R , and T . Line segment BE is divided into four equal parts by points Q , S , and U . The length of PQ equals 3. What is the area of quadrilateral $PQUT$?

- A) $\frac{15}{4}$ B) 4 C) $\frac{17}{4}$ D) $\frac{9}{2}$ E) 5



- A7.** Carry has six cards. On each card is written a positive integer number. Carry chooses three cards and calculates the sum of the three numbers on these cards. Doing this for each of the 20 possible selections of three cards, she finds the following sums: 16 (ten times) and 18 (also ten times).

What is the smallest number occurring on the cards?

- A) 1 B) 2 C) 3 D) 4 E) 5

- A8.** A sequence of numbers starts as 27, 1, 2012, ... The sequence has the following property: the numbers in positions 1, 2, and 3 add up to 2040. The numbers in positions 2, 3, and 4 add up to 2039, the numbers in positions 3, 4, and 5 add up to 2038, and so on. More generally: the numbers in positions k , $k + 1$, and $k + 2$ add up to $2041 - k$.

What number is in position 2013?

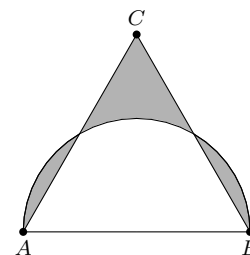
- A) -670 B) -669 C) 670 D) 1341 E) 1342

B-problems

- B1.** Consider all 5-digit numbers. Of these numbers, a have the property that the product of their digits is 25, and b have the property that the product of their digits is 15. Determine $\frac{a}{b}$.

- B2.** An equilateral triangle ABC has sides of length 12. A semicircle with diameter AB intersects sides AC and BC . The two circular segments outside the triangle, and the part of the triangle outside of the circle are coloured gray.

Determine the total gray area.



- B3.** A rectangle consists of a number of cells of a regular graphing paper. The number of these cells that lie at the boundary is the same as the number of cells that do not lie at the boundary. How many cells does the rectangle contain? Give all possible values.

- B4.** For all positive numbers x and y , the operation \triangleleft obeys the following three rules:

Rule 1: $(2x) \triangleleft y = \frac{1}{2} + (x \triangleleft y)$.

Rule 2: $y^2 \triangleleft x = x^2 \triangleleft y$.

Rule 3: $2 \triangleleft 2 = \frac{3}{2}$.

Compute $32 \triangleleft 8$.