

# First Round Dutch Mathematical Olympiad



Friday, Januari 29, 2010

- Time available: 2 hours.
- The A-problems are multiple choice questions. Only one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- The answer to each B-problem is a number. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.  
NOTE: all answers should be given in exact form, like  $\frac{11}{81}$ ,  $2 + \frac{1}{2}\sqrt{5}$  or  $\frac{1}{4}\pi + 1$ .
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- This is a competition, not an exam. Enjoy cracking some unusual mathematical nuts. Good luck!

## A-problems

**A1.** A figure consists of three circles and two lines. What is the maximal number of intersections possible?

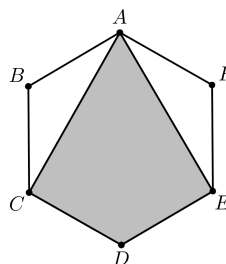
- A) 15      B) 16      C) 18      D) 19      E) 20

**A2.** A test has six questions that are worth 1 to 6 points respectively. When answering a question correctly, the points of that question are added to your score. In case of a wrong answer, the points are subtracted from your score. For example, if only questions 1, 3 and 4 are correctly answered, your score will be  $1 - 2 + 3 + 4 - 5 - 6 = -5$ . How many different values can the final score take?

- A) 20      B) 22      C) 41      D) 43      E) 64

**A3.** The area of a regular hexagon  $ABCDEF$  equals 1. What is the area of the kite  $ACDE$ ?

- A)  $\frac{1}{4}\sqrt{6}$       B)  $\frac{2}{3}$       C)  $\frac{3}{4}$       D)  $\frac{5}{6}$       E)  $\frac{1}{2}\sqrt{3}$



**A4.** Three players play a game using chips. In each round, the player (one of the players) having the largest number of chips, puts one of his chips in the pot and then gives a chip to each of the other players. To start with, the players have 13, 14 and 15 chips, while the pot is empty. The game ends as soon as one of the players has no more chips.

How many chips are in the pot when the game ends?

- A) 36      B) 37      C) 38      D) 39      E) 40

**A5.** Writing down the number  $((((7^6)^5)^4)^3)^2$ , what is the last digit?

- A) 1      B) 3      C) 5      D) 7      E) 9

Please continue on the other side.

**A6.** Calculate  $\left(\left(\sqrt{2} + 1\right)^7 + \left(\sqrt{2} - 1\right)^7\right)^2 - \left(\left(\sqrt{2} + 1\right)^7 - \left(\sqrt{2} - 1\right)^7\right)^2$ .

- A) 2      B) 4      C)  $8\sqrt{2}$       D) 128      E) 512

**A7.** A car's odometer indicates that the car has traveled 2010 miles. The odometer uses six digits and so the display reads 002010. However, in each of the six positions, digit '4' is always skipped and the odometer moves directly from '3' to '5'.

What was the actual number of miles traveled by the car?

- A) 1409      B) 1467      C) 1647      D) 1787      E) 1809

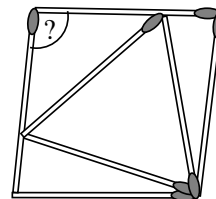
**A8.** Thirty people, no two of which have the same height, are placed in an array of six rows containing five people each. From each row, the shortest person is selected, and from these six, the tallest one is Paul. Also, from each row, the tallest person is selected and among these six James, is shortest. Now we place everyone in a line according to their height: the shortest on the left-hand side and the tallest on the right-hand side.

What place in the line can James *not* be at?

- A) 21 places to the left of Paul      D) 19 places to the right of Paul  
 B) 19 places to the left of Paul      E) 21 places to the right of Paul  
 C) directly next to Paul

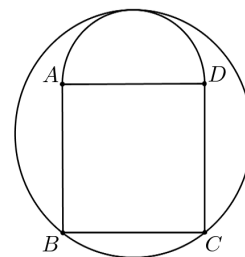
## B-problems

**B1.** Seven matches of equal length touch as indicated in the figure. How many degrees is the marked angle?



**B2.** How many positive integers  $a$  have this property: dividing 2216 by  $a$  leaves a remainder of 29.

**B3.** A figure is composed of a square  $ABCD$  and a semicircle with diameter  $AD$  outside the square. The square has sides of length 1. What is the radius of the circle circumscribing the figure?



**B4.** In an array of 28 rows and 37 columns, a number is written in each position using a red pen: in the top row, from left to right the numbers 1 to 37 are written, in the next row from left to right the numbers 38 to 74 are written, and so on. Using a green pen, we once more write a number in each position, but this time the numbers 1 to 28 are written from top to bottom in the left-most column, in the next column from top to bottom the numbers 29 to 56 are written, and so on.

In the upper left position, the number 1 is written both in red and green. Add all those red numbers, that belong to a position in which the red and green number are equal. What is the resulting sum?